Stochastic Regularities of Electrical Breakdown Initiation in Transformer Oil

A. L. Kupershtokh
Lavrentyev Institute of Hydrodynamics, SB RAS, Novosibirsk, 630090 Russia

I. Vitellas
Public Power Corporation, Athens, Greece

D. P. Agoris
High Voltage Laboratory, University of Patras, Rio, GR 26500, Greece

D. I. Karpov
Lavrentyev Institute of Hydrodynamics, SB RAS, Novosibirsk, 630090 Russia

V. P. Charalambakos
High Voltage Laboratory, University of Patras, Rio, GR 26500, Greece

Abstract- **The experiments on breakdown in transformer oil at small gaps between the hemispherical electrodes under the linearly increasing AC voltage were carried out. The stochastic regularities of breakdown initiation were studied using the theory proposed in [1-5]. Analytical expressions were derived that allowed us to reconstruct probability density function μ(*E*) for transformer oil using the experimental data of present work and [6]. The probability density functions of the breakdown initiation for the transformer oil are determined at various parameters of experiment (radius of curvature of electrodes, gap length between them and the growth rate of the amplitude of AC voltage). The conception of dynamic electric strength originates from stochastic approach proposed by averaging the breakdown voltages over the probability distributions.**

I. Introduction

It is usually implied that ability of a dielectric to maintain the dielectric properties under the action of strong electric fields is characterized by its electric strength. However, it is well known, that average value of electric field, at which breakdown of a dielectric occurs, also depends on specific experimental conditions such as the form and the sizes of electrodes, distance between them, magnitude and the form of applied voltage, etc [6-11]. Therefore, the classical concept about fixed "electric strength" fails. Instead of this, the concept of "dynamic electric strength" of dielectric that depends on the specific conditions listed above have to be used. Well known time-voltage curves are the particular feature of dynamic electric strength.

Moreover, it is well known that the prebreakdown processes in liquid dielectrics have a stochastic nature. Numerous experimental data point to the principal role of stochastic processes at a breakdown in dielectric liquids (for example, statistical time lag, asymmetry and non-reproducibility of streamer detailed structure, tooth-like shape of recordings of current and light pulses, etc). Thus, an adequate description of stochastic regularities of dielectric breakdown has to include probability distribution functions for such processes.

One of the stochastic processes is the initiation of breakdown due to the development of a series of microscopic phenomena at the electrode surface and in a thin dielectric layer contiguous to it. The duration of this stage of breakdown (called the statistical time lag t_S) is a random value for which the probability density depends on the electric field and its distribution along the surface of the electrodes.

Many authors made efforts to describe stochastic regularities of breakdown using various statistical distributions. It is well known the attempts to apply statistics of extreme values [6] and Weibull's distributions [9, 10, 12-17] for interpretation of the numerous available experimental data. Unfortunately, these approaches do not allow one to describe in a simple way how the complete set of experimental conditions (duration and waveform of applied voltage, form and size of electrodes, etc) influences the breakdown.

One of the attempts to investigate the essentially stochastic character of prebreakdown processes in dielectric liquids was made by Lewis in [7]. He proposed to use the distribution function of statistical time lags $f(E)$. This function means the probability density of breakdown initiation in a short time interval.

Researches of late years showed that the probability of breakdown initiation in some local area of dielectric should depends only on magnitude of local electric field in this area and on properties of the substance, provided that pressure and temperature are constant during experiment. These features of breakdown phenomenon allowed us to describe the dynamic electric strength of specific dielectric quantitatively, taking into account essentially stochastic nature of breakdown.

In 1993 it was proposed that the basic stochastic processes of streamer inception at the electrode could be described by macroscopic function $\mu(E)$ [1-3]. This function is the probability density of breakdown initiation in a short time interval at a small element of an electrode surface near which the electric-field value equals to E. The function $\mu(E)$ increases sharply with increase in the electric field. The parameters of the function $\mu(E)$ depend on the properties of a specific dielectric and perhaps on the material of the electrodes. For a specific dielectric, the function $\mu(E)$ defines also its dynamic electric strength. It is obvious that function introduced in [7] $f(E) = \int_S \mu(E)\, ds$, where the domain of integration is the entire electrode area.

This macroscopic approach allows one to obtain the dependencies of the breakdown initiation probability in time on

0-7803-7114-3/01/$10.00 (C) 2001

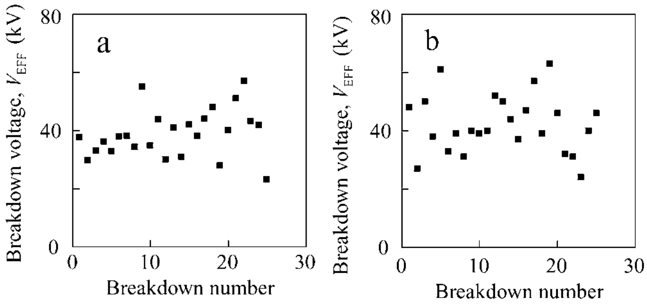

Fig. 1. Typical series of breakdowns in transformer oil under AC voltage of linearly increasing amplitude. Gap length between stainless steel electrodes was d = 1.7 (a) and 2.5 (b) mm.

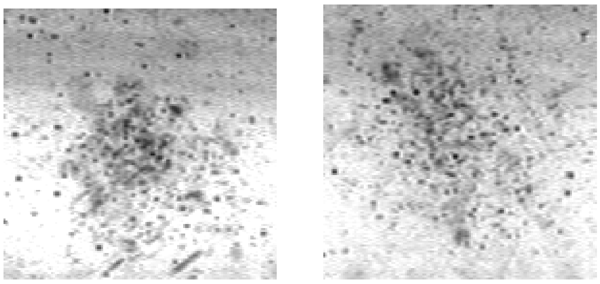

Fig. 2. Photos of the pitting on surface of the stainless steel electrodes after series of breakdown in transformer oil. Areas of size 3×3 mm are shown. R = 19 mm. d = 0.83 mm (a) and 2.5 mm (b).

the applied voltage, its waveform, electrode area, and gap length and to simulate the breakdown, including its stochastic features. And vice versa, it is possible to reconstruct the function μ(E) from experimental data. In the simplest case of DC voltage and flat electrodes

$$\mu(E) = \left(\langle t_S \rangle \cdot S\right)^{-1} \tag{1}$$

where S is the area of the electrode.

In the present work the macroscopic approach developed for AC voltage and hemispherical electrodes was applied to the data on breakdown in transformer oil. The stochastic theory of inception of breakdown in liquid dielectrics under increasing AC voltage was developed.

II. Experiments

The experiments on breakdown in synthetic transformer oil "TECHNOL 2002 (ISO 9001)" were carried out. A new pair of polished spherical stainless steel electrodes with surface radius R = 19 mm were used in each series of experiments. The gap lengths between the electrodes d were in the range from 0.5 to 2.5 mm. High voltage tests were carried out using the standard generator "Baur A-6832". The amplitude of AC voltage of frequency 60 Hz increased with a constant rate.

In experiments, the current effective value of voltage V_{EFF} at which breakdown of a dielectric occurred was registered (Fig. 1). A built-in electronic device removed the voltage from the electrodes immediately after breakdown. The rate of increase of effective value of applied voltage k_e = 0.5, 1, 3 kV/s was switched over after each breakdown. Thus, three data sets of breakdown voltages were obtained in one series of experiments under identical conditions. Period between breakdowns was approximately 3 minutes. The conditioning effect was observed in every series of breakdowns. We took into account only the breakdowns after first 45 shots in series. The results of six series of breakdowns are shown in the Table I.

TABLE I.

N	d, mm	k_e, kV/s	N_0	$<V_{EFF}>$, kV	$<E_0>$, kV/cm	V^*_{EFF}, kV	E^*_0, kV/cm
1	2.5	0.5	60	50.6	286	53	300
	2.5	1	60	55.5	314	58	328
	2.5	3	60	64.0	362	71	402
2	1.0	0.5	40	24.1	341	25	354
	1.0	1	40	24.6	348	25.5	361
	1.0	3	39	29.8	421	34	481
3	0.5	0.5	48	20.5	580	21.5	608
	0.5	1	50	22.4	634	24	679
	0.5	3	50	23.8	673	25	707
4	0.83	0.5	27	25.7	438	30	511
	0.83	1	27	29.1	496	31	528
	0.83	3	27	27.6	470	30.5	519
5	1.66	0.5	25	32.6	278	36	307
	1.66	1	25	38.9	331	42	358
	1.66	3	25	45.8	390	49	417
6	2.5	0.5	25	42.2	238	46	260
	2.5	1	25	46.7	264	49	277
	2.5	3	25	57.0	322	61	345

Here N is series number, N_0 is the number of breakdowns, $< V_{EFF} >$ is the average effective value of breakdown voltage, $< E_0 >$ is the corresponding value of amplitude of electric field averaged along an axis between electrodes, V^*_{EFF} is the effective value of a voltage at which in a series of experiments breakdown occurred with the fixed probability $P_+(t) = 0.63$, E^*_0 is the corresponding amplitude value of average electric field along an axis between electrodes.

Two typical distributions of places of breakdown initiation on the surfaces of the electrodes corresponding to the series 4 and 6 are shown in the Fig. 2 (breakdown pitting). Total numbers of breakdowns are approximately equal to 126 in the series 4 and 120 in the series 6. It is clearly seen that characteristic size of pitting region increases with the gap spacing.

III. Calculation of the Electric-Field Distribution Between Hemispherical Electrodes

A good approximation for electric field strength on the surface of hemispherical electrodes is given by field distribution in the gap between two metallic spheres (Fig. 3). Here V is the applied voltage, R is the radius of spherical electrodes, and d is the gap length between them. The electric field was obtained analytically by solving the Laplace equation in bispherical coordinates and is expressed by the formula:

0-7803-7114-3/01/$10.00 (C) 2001

$$E(\xi,\eta)=\frac{E_0\, d\sqrt{2(\mathrm{ch}\xi-\cos\eta)}}{2R\,\mathrm{sh}\,\xi_1}\sum_{l=0}^{\infty}\frac{e^{-\left(l+\frac{1}{2}\right)\xi_1}\,\mathrm{P}_l(\cos\eta)}{\mathrm{sh}((2l+1)\,\xi_1)}\times \tag{2}$$
$$\left\{\mathrm{sh}\left(\left(l+\frac{1}{2}\right)(\xi+\xi_1)\right)\mathrm{sh}\,\xi+2(\mathrm{ch}\,\xi-\cos\eta)\left(l+\frac{1}{2}\right)\mathrm{ch}\left(\left(l+\frac{1}{2}\right)(\xi+\xi_1)\right)\right\}.$$

Here $E_0 = V/d$ is an average electric field strength along an axis between electrodes, ξ and η are bispherical coordinates (electric field potential and electric strength do not depend on azimuthal angle α because of axial symmetry of the problem, $-\xi_1 < \xi < \xi_1$, $0 < \eta < \pi$), P_l is the Legendre polynomial of index l, $\xi_1 = \ln\left(1+\beta+\sqrt{\beta(2+\beta)}\right)$, $\beta = d/2R$ is a relative length of the gap between identical electrodes. The relation between bispherical coordinate η and polar angle θ on the sphere counted from the symmetry axis is given by the expression $\cos\eta = \dfrac{1-(1+\beta)\cos\theta}{1+\beta-\cos\theta}$.

For a quasi-uniform field in a narrow gap between spherical electrodes, the electric field strength changes weakly along electric lines of force. Therefore, one can consider that $E \approx V/l$ (see Fig. 3) and the following approximate formula is valid [3]

$$E \approx \frac{E_0}{1+(1-\cos\theta)/\beta} \ . \tag{3}$$

The plot of a relative electric field E/E_0 at electrode surface versus θ is given in a Fig. 4. The direction $\theta = 0$ corresponds to the maximum value of electric field at the apex. Only a small part of the electrode area near the symmetry axis makes a major contribution to breakdown inception because of the sharp dependence of the function $\mu(E)$ on the electric field. For this region, the approximate formula (3) (Fig. 4, curve *2*) practically coincides with the exact solution (2) (Fig. 4, curve *1*). For example, the difference between them is less than 2% of the maximum field strength for a gap length corresponding to $\beta = 0.02$ [3].

It is important that electric field is not constant along the electric field lines. The distribution of electric field strength along the symmetry axis of system of two spherical electrodes is shown in Fig. 5. The coordinate z along symmetry axis is expressed through bispherical coordinate ξ as $z = \dfrac{R\,\mathrm{sh}\,\xi_1\,\mathrm{sh}\,\xi}{\mathrm{ch}\,\xi+1}$. The maximum value of electric field on the surface of electrodes is somewhat higher than the average value along the axis of symmetry E_0. Factor of amplification of electric field on the surface of electrode in comparison with the average value along the axis $a(\beta) = E_{max}/E_0$ does depend on the parameter β (Table II). Introduction of correction factor $a(\beta)$ gives the possibility to considerably extend the range of the applicability of formula (3) to the values $\beta \approx 0.1$, if we use a value of $a(\beta)E_0$ instead of E_0 .

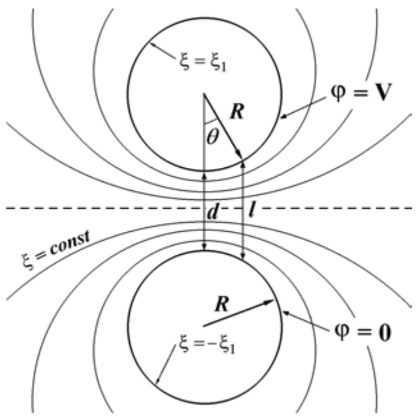

Fig. 3. The configuration of spherical electrodes and the surfaces of equal electric field potential ξ = const.

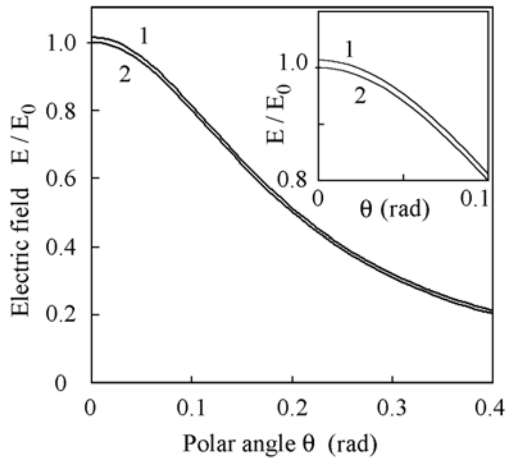

Fig. 4. Electric field distribution along the surface of a spherical electrode for $\beta = 0.02$. Exact solution (2) is shown by curve *1* and approximate solution (3) is shown by curve *2*.

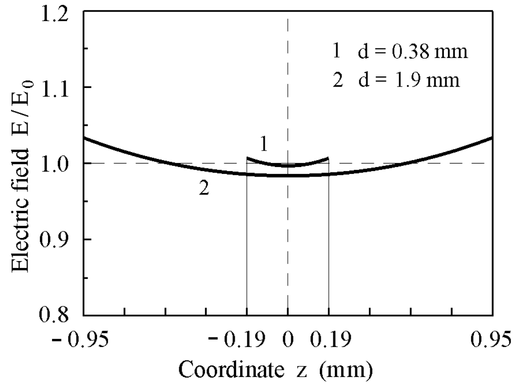

Fig. 5. Distributions of electric field strength along the symmetry axis of two spherical electrodes obtained from the exact solution (2) at $\beta = 0.01$ (curve *1*) and $\beta = 0.05$ (curve *2*). R = 19 mm.

TABLE II.

β	$a(\beta)$
0.01	1.007
0.05	1.034
0.066	1.044
0.1	1.068
0.2	1.13

IV. MACROSCOPIC APPROACH TO BREAKDOWN INITIATION

A. Probability of Breakdown Initiation

In stochastic approach offered earlier in [1-5], macroscopic function μ(E) was introduced which depends on local electric field. It was supposed that probability of breakdown inception near small element of surface of electrode at time t does not depend on previous moments of time and does not depend on events near other elements of electrode [2]. The function μ(E) has physical sense of probability density of breakdown initiation on a small element of electrode surface in a short interval of time. The probability of breakdown inception in time t is equal to

$$P_+(t) = 1 - \exp(-H) \quad (4)$$

where the value of integral of electric field action expresses through the function μ(E) and changes in time as

$$H(t) = \int_0^t \left(\int_S \mu(E)\, ds \right) dt . \quad (5)$$

For example, $H(t) = S\int_0^t \mu(E)\, dt$ for flat electrodes and

$$P_+(t) = 1 - \exp\left(- S\int_0^t \mu(E)\, dt \right).$$

For hemispherical electrodes with a small gap distance between them, it is possible to turn the integration in (5) from integral over the surface of electrode to the integral over electric field, using the approximation (3) [1, 3]

$$\int_S \mu(E)\, ds \approx d\pi R E_0 \int_0^{E_0} \frac{\mu(E)}{E^2} dE \quad . \quad (6)$$

In the right side of the equation (6) we used zero as lower limit of integration, bearing in mind the sharp dependence of function μ(E) on electric field. Using (6), we introduced in [1] for the small gaps the concept of the effective area of hemispherical electrodes in accordance with the formula:

$$S_* = \frac{d\pi R E_0 \int_0^{E_0} \frac{\mu(E)}{E^2} dE}{\mu(E_0)} . \quad (7)$$

It means that any probabilities $P_+(E)$ plotted for flat electrodes could be used also for hemispherical electrodes in the case of narrow gaps (β < 0.1) if instead the value S we imply the effective area S_*. For example, for hemispherical electrodes from (4), (5) and (7) one can obtain the expression for probability of breakdown under stepwise voltage pulse in the form

$$P_+(t) = 1 - \exp\left(-tS_*\mu(E_0)\right) = 1 - \exp\left(- td\pi R E_0 \int_0^{E_0} \frac{\mu(E)}{E^2} dE \right).$$

The effect of an increase in the area, on which the breakdown originated, with an increase in the product of electrode radius and gap spacing Rd is well known from the experiment [8]. The formula for effective electrode area $S_* = d\pi Rk$ obtained from simple geometric relations is usually used in electrical engineering [11]. Here $k = \Delta E/(E_0 - \Delta E)$ where ΔE is the permissible field deviation from maximum field at apex of the sphere. The relevant value of k is some undefined value that depends on the particular liquid. In our work, the value S_* introduced depends on features of particular dielectric through the function μ(E) and also generally on electric field E_0. In the particular case, if it is possible to approximate the function μ(E) for dielectric by the power-law dependence $\mu(E) = A(E/E_1)^n$, one can obtain $S_* = \frac{d\pi R}{n-1}$. For example, we have $S_* \approx 0.79\, d\, R$ if use the value of parameter n equal to 5.

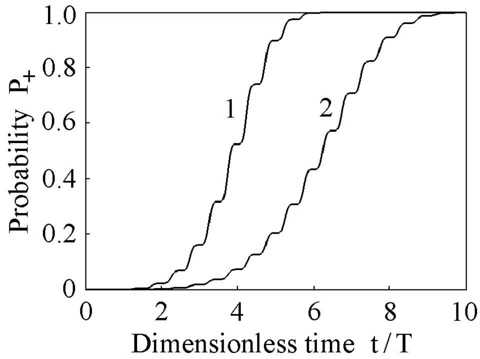

Fig. 6. The probability of breakdown initiation for flat electrodes vs. t/T, where T is the period of AC voltage. $C = 10^{-6}$ (curve 1), 10^{-7} (curve 2), $n = 4$.

B. Macroscopic Theory of Breakdown at AC Voltage

In the case of AC voltage with linearly increasing amplitude, the current value of voltage is $V = \sqrt{2} k_e t \sin(\omega t)$. Using the power approximation $\mu(E) = AE^n$ for the function μ(E) for flat electrodes of area S with the gap distance d, we obtained the expression

$$H(t) = C \int_0^{\omega t} z^n\, |\sin(z)|^n\, dz$$

where $C = \frac{AS(\sqrt{2}k_e)^n}{d^n \omega^{n+1}}$. The plot of P_+ versus the dimensionless time t/T is shown in Fig. 6.

At AC voltage of slowly increasing amplitude, the product $k_e t$ changes only slightly during each half-cycle and the form of every voltage pulse is practically proportional to $\sin(\omega t)$. In this case, the action integral changes over a half-cycle by the value

$$\Delta H_i = \frac{SAE_i^n T}{2\pi} \int_0^{\pi} \sin^n(z)\, dz ,$$

where E_i is the amplitude of the electric field when the number of voltage half-cycles is equal to i.

When breakdown occurs after many voltage half-cycles, we have

$$H_i = \sum_{j=1}^{i} \Delta H_j \approx \frac{SdAE_i^{n+1}}{\sqrt{2}k_e \pi(n+1)} \int_0^{\pi} \sin^n(z)\,dz\,. \qquad (8)$$

When large set of data on breakdowns in each series of experiments ($N_0 >> 100$) is available, it is possible to reconstruct values of $\mu(E)$ using the histograms of breakdown voltages measured in experiments. From (8) we obtained the formula that expresses the values of $\mu(E)$ in terms of the experimental distribution of breakdown voltages:

$$\mu(E) = \frac{\pi\sqrt{2}k_e \ln(N_i / N_{i+l})}{Sd\,\Delta E_l \int_0^{\pi} \sin^n(z)\,dz}\,. \qquad (9)$$

Here N_i and N_{i+l} are the numbers of breakdowns in the series that occurred not earlier than the i and (i+l) voltage half-cycles, respectively, after the voltage was switched on, ΔE_l is the increment of the electric-field strength over l half-cycles (it was assumed that $l << i$).

Using the approximations (3) and (6) we also obtained the following approximate formula for hemispherical electrodes with a small interelectrode gap:

$$\Delta H_i = \frac{Rd\,AE_{i0}^n\,T}{2(n-1)} \int_0^{\pi} \sin^n(z)\,dz\,.$$

Here E_{i0} is the amplitude of the average electric field on the axis between the electrodes. By analogy,

$$H_i = \frac{Rd^2 AE_{i0}^{n+1}}{\sqrt{2}k_e(n^2-1)} \int_0^{\pi} \sin^n(z)\,dz\,. \qquad (10)$$

Thus, we have the dependencies of the probability of breakdown on the main parameters such as the radius of the electrode surface (or electrode area in the case of flat electrodes), gap distance, rate of increase in voltage, etc. It is interesting, that the current value of electric strength depends only on parameter $b = k_e\pi/(Sd)$ where $S = \pi Rd/(n-1)$ for hemispherical electrodes. This parameter is convenient for preliminary comparing of the experimental data obtained at different geometry of electrodes and different values of k_e, and S (for flat electrodes) or d and R (for hemispherical electrodes). For example, the results of experiments given in the Table I were plotted as the dependencies on the parameter b (Fig. 7).

From (10) it is easy to derive the formula for reconstruction of $\mu(E)$ from data on experimental distribution of breakdown voltages that valid for hemispherical electrodes:

$$\mu(E) = \frac{\sqrt{2}k_e(n-1)\ln(N_i / N_{i+l})}{Rd^2\,\Delta E_l \int_0^{\pi} \sin^n(z)\,dz}\,. \qquad (11)$$

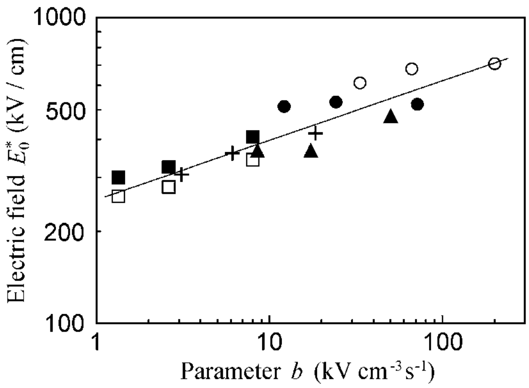

Fig. 7. Dependence E^*_0 on parameter b for transformer oil. d = 2.5 (□,■), 1.66 (+), 1.0 (▲), 0.83 (●), 0.5 (○) mm.

Another way to determine the parameters of function $\mu(E)$ from experiments is to use (8) or (10) with fixed values of H that correspond some fixed probability of breakdown P_+. It is convenient to use the values of electric field E^*_0 corresponding the values H = 1, for which the probability of breakdown $P_+ = 0.63$.

This method could be applicable only for a large enough series of breakdowns. Otherwise, significant statistical variations of breakdown voltage (Fig. 1) will result in big uncertainties in the values of E^*_0 and, consequently, the reconstructed values of $\mu(E)$.

At the same time, mean value of amplitude of an electric field of breakdown $<E_0>$ could be determined from the same series of breakdown with smaller statistical error. In case of power-law approximation for $\mu(E)$, the explicit analytical expressions for determination of values of $\mu(E)$ could be obtained from probability distribution (4) and (5). For example, for hemispherical electrodes at small gap length the analytical formula for reconstruction of values of function $\mu(E)$ was obtained using (10)

$$\mu(<E_0>)\frac{F(n)}{n-1} = \frac{\sqrt{2}\,k_e}{\pi d^2 R <E_0>}$$ where the function

$$F(n) = \frac{\int_0^{\pi} \sin^n(z)\,dz}{\pi(n+1)\,\Gamma\left(\frac{n+2}{n+1}\right)^{n+1}}$$

depends only on an exponent n in the approximation of $\mu(E)$. The mean value of electric strength depends also only on parameter b mentioned above.

However, power-law approximation of function $\mu(E)$ gives sometimes too weak dependence on an electric field. In general case of arbitrary form of function $\mu(E)$ for hemispherical electrodes we have

$$H(t) = d\pi R \int_0^t \left(E_0 \int_0^{E_0} \frac{\mu(E)}{E^2} dE \right) dt \qquad (12)$$

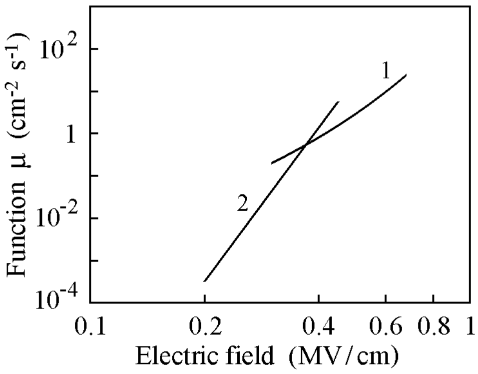

Fig. 8. Values of function μ(E) reconstructed from experiment. Curve *1* is approximation in the form (13) for transformer oil "TECHNOL 2002 (ISO 9001)". Straight line *2* is the function μ(E) reconstructed for transformer oil from the data of [6].

where $E_0(t) = \sqrt{2}k_e t \sin(\omega t)/d$. The approximation in the form

$$\mu = AE^2 \exp(E/g) \quad (13)$$

is sharper than power-law dependence and describes the histograms of breakdown voltages better. This approximation also allows one to calculate the integral in (12) over electric field analytically. Integration over time in (12) was carried out numerically right up to the moment corresponding to amplitude value of electric field E^*_0. Then, the parameters A and g were obtained using the condition $H = 1$ ($P_+ = 0.63$) for each series of breakdown. We used six values of E^*_0 obtained in the series 1 and 3 (Table I) in which the number of breakdowns after conditioning period was approximately equal to or greater than 50. The values of function μ(E) reconstructed from data of present work are shown in Fig. 8 (curves *1*).

The experimental data of Weber and Endicott [6] on breakdown in transformer oil under AC voltage with a frequency of 60 Hz were also analyzed using the above mentioned approach (9) for flat electrodes. The effective value of

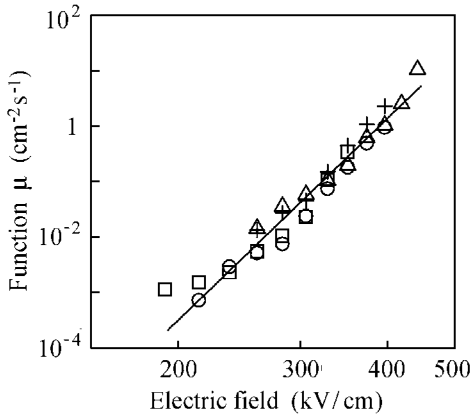

Fig. 9. Reconstructed values of function μ(E) for transformer oil from the data of [6]. Pairs of flat brass electrodes of area S = 1.54 (Δ), 4.9 (+), 15 (o), 29 (□) cm^2 were used. N_0 = 400 for each pair of electrodes.

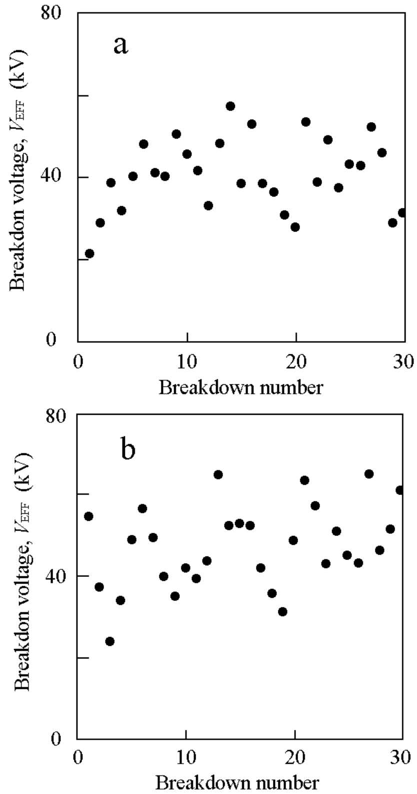

Fig. 10. Computer simulation of series of breakdown. (a) d = 1.66 mm, k_e = 1 kV/s. (b) d = 2.5 mm, k_e = 0.5 kV/s.

the applied voltage increased with a constant rate k_e = 3 kV/s. In these experiments, four pairs of flat brass electrodes of areas S = 1.54, 4.9, 15, and 29 cm^2 were used at a gap spacing d = 0.19 cm. The results of reconstruction of the function μ(E) obtained from (9) for each pair of electrodes are shown in Fig. 8, curve *2* and Fig. 9.

V. Stochastic Simulation of Breakdown Initiation

Using the reconstructed function μ(E), one can plot any dependencies of the breakdown initiation probability for various geometry of electrodes and also for various magnitude, duration, and waveform of the applied voltage.

Within the framework of the stochastic approach proposed, computer simulation of series of breakdowns was carried out using (4), (12), and (13).

Statistical time lag before breakdown t_S was determined from

$$\int_0^{\omega t_S} \tau|\sin\tau|\left[e^{B\tau|\sin\tau|} - 1\right] d\tau = -\frac{\omega^2}{\sqrt{2}\pi AgkR}\ln(\zeta), \quad (14)$$

where $B = \sqrt{2}k/(\omega d g)$, ζ is a random number that is uniformly distributed in the interval from 0 to 1. The integration on the left hand of (14) was carried out numerically until the value of the integral was equal to the value of expression on the right hand.

Typical series of breakdown in transformer oil obtained in computer simulations are shown in Fig. 10. These results are in good agreement with the experimental results (Fig. 1). It is also possible to carry out the simulations of breakdown pitting on the surface of hemispherical electrodes [4, 5].

VI. Conclusions

The proposed approach allowed us to obtain the explicit dependences of probability of breakdown inception on electric-field strength for flat and hemispherical electrodes under AC voltage of linearly increasing amplitude.

It is shown that the effective area of hemispherical electrodes in the case of narrow gaps between them is proportional to the product of the radius of electrode to the gap length. The coefficient depends on features of particular dielectric through the function $\mu(E)$ and in general case depends on magnitude of electric field.

Within the framework of the stochastic approach several new analytical dependences for distribution of probabilities of breakdown initiation were obtained at changes of geometry of an inter-electrode gap (S or R and d), and also of rate of increase of AC voltage. All these results obtained for flat electrodes are also valid for coaxial cylindrical electrodes provided that edge effects are negligible too.

Usually the experimental data are fitted in logarithm-logarithm scale by Weibull functions (on electric field, area of electrodes and time) [9, 10, 12-17]. In some sense our probability distribution (4) with function $H(t)$ in form (5) is more general. For power-law approximation of $\mu(E)$ we can obtain Weibull-like distributions from (4) and (8) for flat or (10) for hemispherical electrodes. In this case, it depends on two parameters in power-law approximation of function on electric field $\mu(E)$. Hence, the Weibull distributions in time, size, and electric field strength in some sense are close to the particular forms of our distribution.

The opportunity of direct stochastic computer simulations of experiments on breakdown in dielectric liquids is demonstrated. The proposed approach describes essentially stochastic nature of breakdown that is necessary to take into account at designing electrotechnical devices in which liquid dielectrics are employed.

Acknowledgments

This work was supported in part by the Siberian Branch of the Russian Academy of Science (grant No. 47, "Development of scientific basis for designing a new generation of environmentally appropriate power equipment using perfluorocarbon liquids with the given properties"). This work was supported in part also under a grant of the NATO Science Fellowship Program – 2000 and 2001.

References

[1] V. F. Klimkin, A. L. Kupershtokh, "Statistical lag time in fluctuation model of liquid dielectric breakdown and experimental results," Proc. 11th Int. Conf. on Conduction and Breakdown in Dielectric Liquids, Baden-Dättwil, Switzerland, pp. 395–399, 1993.

[2] B. G. Vainer, A. L. Kupershtokh, "Measurements of statistical lag time of breakdown in thin amorphous layers of SiO_2", Conf. Record of the 1998 IEEE Int. Symp. on Electrical Insulation, IEEE No. 98CH36239, Arlington, USA, pp. 169–172, 1998.

[3] A. L. Kupershtokh, D. I. Karpov, "Stochastic Features of Initiation of Liquid Dielectric Breakdown at Small Area of Positive Electrode," Proc. 13th Int. Conf. on Dielectric Liquids, Nara, Japan, pp. 203–206, 1999.

[4] A. L. Kupershtokh, E. I. Palchikov, D. I. Karpov, A. P. Ershov, "Probability Density Function of Electrical Breakdown Initiation in Dielectric Liquids under AC and DC Voltage," Proc. 2nd Int. Workshop on Electrical Conduction, Convection, and Breakdown in Fluids, Grenoble, France, pp. 91–94, 2000.

[5] A. L. Kupershtokh, E. I. Palchikov, D. I. Karpov, A. P. Ershov, "Stochastic Regularities of Electrical Breakdown Initiation in Dielectric Liquids under AC Voltage," Proc. of the 6th Int. Conf. on Modern Problems of Electrophysics and Electrohydrodynamics of Liquids, St. Petersburg, Russia, pp. 175–178, 2000.

[6] K. H. Weber, H. S. Endicott, "Area effect and its extremal basis for the electric breakdown of transformer oil," Trans. of the Amer. Institute of Electrical Engineers, vol. 75, pp. 371–381, 1956.

[7] T. J. Lewis and B. W. Ward, "A statistical interpretation of the electrical breakdown of liquid dielectrics", Proc. Roy. Soc. A: Mathematical and Physical Sciences, Vol. 269, No. 1337, pp. 109-124, 1962.

[8] J. Gerhold, M. Hubmann, E. Telser, "Gap size effect on liquid helium breakdown," Cryogenics, vol. 34, pp. 579–586, 1994.

[9] H. Goshima, N. Hayakawa, M. Hikita, H. Okubo, K. Uchida, "Weibull statistical analysis of area and volume effects on the breakdown strength in liquid nitrogen," IEEE Trans. on Diel. and Electr. Insul., vol. 2, No. 3, pp. 385–393, 1995.

[10] J. Suehiro, K. Ohno, T. Takahashi, M. Miyama, M. Hara, "Statistical characteristics of electrical breakdown in saturated superfluid helium", Proc. of the 12th Int. Conf. on Conduction and Breakdown in Dielectric Liquids, IEEE No. 96CH35981, Roma, Italy, pp. 320-323, 1996.

[11] W. F. Gauster, "Über Oberflächeneffekte beim elektrischen Durchbruch von Flüssigkeiten", Österreichisches Ingenieur-Archiv, vol. 10, pp. 160–167, 1956.

[12] R. M. Hill, L. A. Dissado, "Theoretical basis for the statistics of dielectric breakdown," J. Phys. C: Solid State Phys., vol. 16, pp. 2145–2156, 1983.

[13] L. A. Dissado, J. C. Forthergill, S. V. Wolfe, R. M. Hill, "Weibull statistics in dielectric breakdown: Theoretical basis, applications and implications," IEEE Trans. on Electr. Insul., vol. 19, No. 3, pp. 227–233, June 1984.

[14] L. A. Dissado, "Theoretical basis for the statistics of dielectric breakdown," J. Phys. D: Appl. Phys., vol. 23, pp. 1582–1591, 1990.

[15] J. Gerhold, M. Hubmann, E. Telser, "About the size effect in LHe-breakdown", Proc. of the 12th Int. Conf. on Conduction and Breakdown in Dielectric Liquids, IEEE No. 96CH35981, Roma, Italy, pp. 324-328, 1996.

[16] J. Gerhold, M. Hubmann, E. Telser, "Breakdown probability and size effect in liquid helium", IEEE Trans. on Diel. and Electr. Insul., vol. 5, No. 3, pp. 321–333, 1998.

[17] J. Gerhold, "Cryogenic liquids – a prospective insulation basis for future power equipment", Proc. of the 13th Int. Conf. on Dielectric Liquids, IEEE No. 99CH36213, Nara, Japan, pp. 365-371, 1999.

0-7803-7114-3/01/$10.00 (C) 2001