

# Simulation of waves of partial discharges in a chain of gas inclusions located in condensed dielectrics

A L Kupershtokh<sup>1</sup> and D I Karpov<sup>1</sup>

<sup>1</sup> Lavrentyev Institute of Hydrodynamics SB RAS, 15 Lavrentyev Prosp., Novosibirsk, 630090, Russia

E-mail: [skn@hydro.nsc.ru](mailto:skn@hydro.nsc.ru)

**Abstract.** A stochastic model of partial discharges inside gas inclusions in condensed dielectrics was developed. The possibility of a "relay-race" wave propagation mechanism of partial discharges in a linear chain of gas inclusions is shown. The lattice Boltzmann method is successfully implemented for three-dimensional computer simulations of flows of dielectric fluid with bubbles. Growth and elongation of bubbles in a liquid dielectric under the action of a strong electric field are simulated. The physical model of propagation of partial discharges along a chain of gas bubbles in a liquid is formulated.

## 1. Introduction

When the high voltage is applied to an electrode gap filled with a dielectric, partial discharges may occur that do not lead to the loss of the insulating properties of the dielectric as a whole. At this time, the pulses of current are registered in the discharge circuit, the number and intensity of which can predict the probability of a breakdown of the gap. One type of partial discharge in condensed dielectrics are electrical discharges inside the cavities and bubbles, which have dimensions of the order of 10 microns. The works [1-3] are devoted to computer simulations of partial discharges in liquid and solid dielectrics. In this paper, the partial discharges in condensed dielectric between two flat electrodes are studied. Gas inside the small cavities in solid dielectrics or inside the microbubbles in a liquid has a dielectric strength which is much lower than that of the condensed matter. The possibility of the generation of "relay-race" wave propagation of partial discharges in the chain of inclusions is demonstrated. This mechanism differs from hopping spread streamers in the medium with bubbles [4] since for "relay-race" mechanism streamers do not occur in the condensed phase and thus there is no breakdown of the gap. Moreover, the stochastic nature of the phenomenon is taken into account.

## 2. Partial discharges in a chain of gas cavities located in solid dielectrics

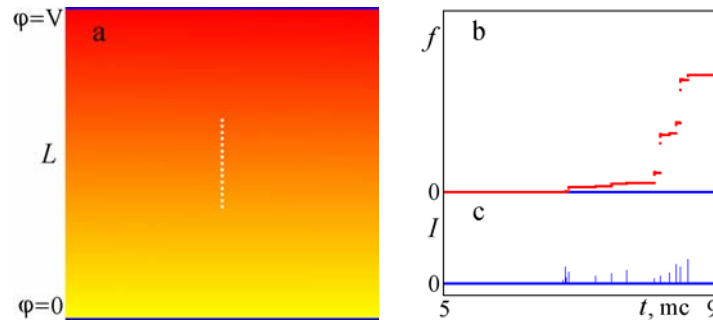
At the same dimensions of cavities in the bulk of the dielectric and at the same gas pressures inside them, the probability of microbreakdowns in inclusions depends on the local electric field within them  $E_i$ . The stochastic criterion MESTL [5] was used to describe the occurrence of micro-discharges in cavities. For all nonconducting cavities, the stochastic lag times  $t_i$  were calculated in accordance with the density distribution function for the probability of rare events

$$F(t_i) = r(E_i) \exp(-r(E_i)t_i). \quad (1)$$



Here,  $r(E)$  is a sharply increasing function of the electric field. For small time step  $\Delta t \ll 1/r(E)$ , the probability of a micro-breakdown in a cavity is  $f \approx r(E)\Delta t$ . It was assumed that the micro-breakdown in a cavity occurs at the current time step if the condition  $t_i < \Delta t$  is satisfied.

A chain of several identical gas inclusions placed along the electric field lines was studied (figure 1a). The distance between the inclusions is constant  $\Delta y$ . The computational lattice spacing is  $h$ . DC voltage  $V$  sufficient for the occurrence of partial discharges was applied to the electrodes.



**Figure 1.** (a) – Chain of gas cavities in the solid dielectric.  $L = 1$  mm. Lattice is  $200 \times 200$ .  $\Delta y = 4h$ . (b) – Increase of the probability of a microdischarge in one of the cavities with each discharge in other cavities.  $\alpha = 0.005$  1/(V·ms). (c) – Current  $I$  in the circuit induced by partial discharges.

The potential of the electric field in the region between the electrodes (figure 1a) is calculated numerically at each time step by solving the Poisson's equation together with the equations of the electric charge transfer

$$\operatorname{div}(\varepsilon \nabla \varphi) = -4\pi q, \quad \mathbf{E} = -\nabla \varphi, \quad (2)$$

$$\mathbf{j} = \sigma \cdot \mathbf{E}, \quad \frac{\partial q}{\partial t} = -\operatorname{div} \mathbf{j}. \quad (3)$$

Here,  $\varepsilon$  is the permittivity of fluid,  $q$  is the electric charge density. The periodic boundary conditions are used in  $x$  direction. It was assumed that current density  $\mathbf{j}$  and conductivity  $\sigma$  are nonzero only within gas inclusions after their breakdown. The permittivity of the condensed dielectric is  $\varepsilon = 2$ .

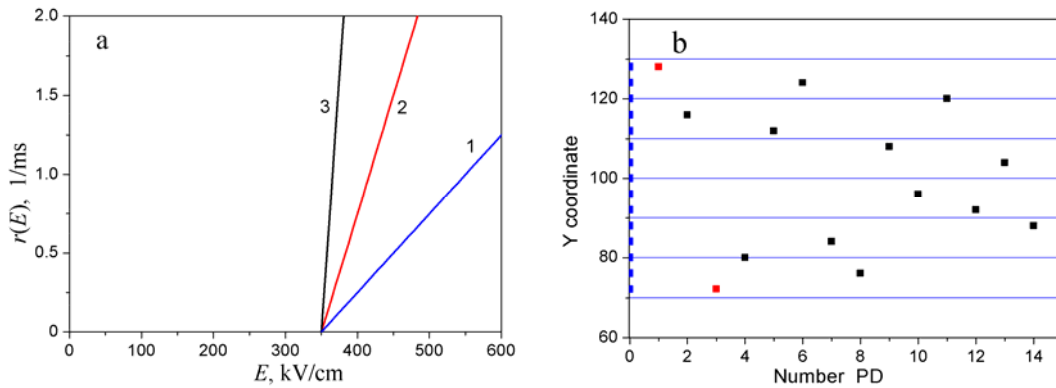
Initially, the electric field is somewhat larger inside the inclusions that are at the edges of the chain than inside the inner inclusions. However, it is lower than the field inside single isolated cavity. After the breakdown inside one of the gas inclusions, the electric field and, accordingly, the probability  $f$  of a breakdown inside the neighbor inclusions during time step  $\Delta t$  increase significantly (figure 1b). The dependences of the function  $r(E)$  for the occurrence of partial discharges in the cavity are shown in figure 2a that take into account the threshold character of partial discharges [1]

$$r(E) = \begin{cases} 0 & \text{при } E \leq E_*, \\ \alpha(E - E_*) & \text{при } E > E_*. \end{cases} \quad (4)$$

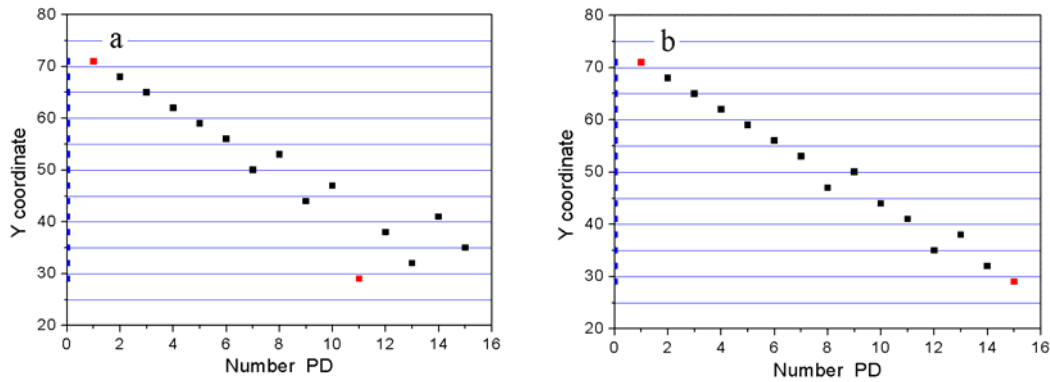
Here,  $E_*$  is the minimum electric field at which partial discharges in the cavity are possible. For inclusions  $d \sim 10$  microns, the breakdown voltage of air in a cavity is  $\approx 350$  V [6], which corresponds to the threshold field  $E_* \approx 350$  kV/cm.

The sequence of partial discharges in the inclusions has a stochastic character at a relatively weak dependence  $r(E)$  or at a weak mutual influence of cavities on each other (figure 2b).

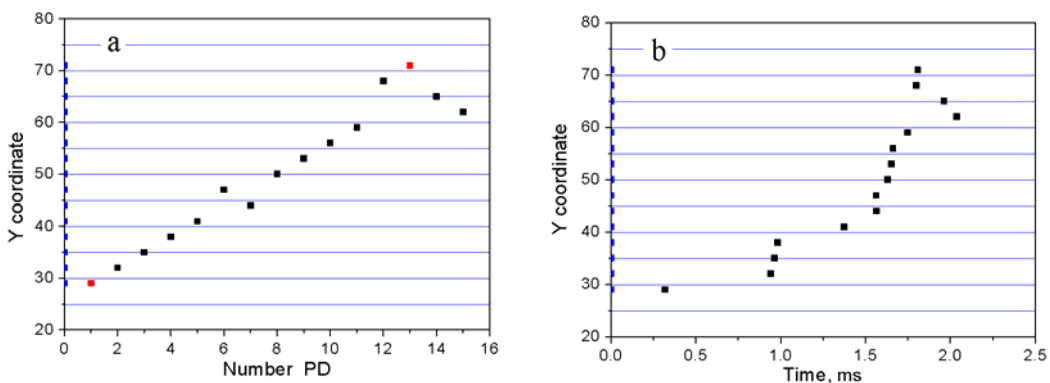
If the dependence  $r(E)$  is sharper, then, under certain conditions, there is an interesting phenomenon – a "relay-race" mechanism of propagation of partial discharges in the chain of insulation defects. Such waves of partial discharge are shown in figures 3 and 4a.



**Figure 2.** (a) – The probability of the first micro discharge in the cavity  $d = 10$  microns in size.  $\alpha = 0.005$  (curve 1),  $0.015$  (curve 2),  $0.065$  (curve 3) 1/(V·ms),  $E_* = 350$  kV/cm. (b) – Partial discharges in the chain of cavities at  $\Delta y = 4h$  and  $\alpha = 0.005$  1/(V·ms). The initial positions of the inclusions are shown at the left by blue.



**Figure 3.** "Relay-race" mechanism of the propagation of a wave of partial discharges along the chain of gas inclusions.  $\alpha = 0.015$  (a),  $0.065$  (b) 1/(V·ms). Lattice is  $100 \times 100$ .  $\Delta y = 3h$ .



**Figure 4.** (a) – Wave of partial discharges.  $\alpha = 0.065$  1/(V·ms). (b) – The sequence of partial discharges in time. Lattice is  $100 \times 100$ .  $\Delta y = 3h$ .

As the partial discharges propagate along the chain of cavities, the electric field increases inside the remaining cavities. At the same time, the statistical time lag of a micro-breakdown decreases, and the average velocity of wave propagation increases (figure 4b).

### 3. The lattice Boltzmann method

The lattice Boltzmann method (LBM) is successfully implemented for three-dimensional computer simulations of liquid dielectric flows. The lattice Boltzmann method describes the viscous flows of fluids with an arbitrary equation of state and simulates the interfaces between vapor and liquid phases with a surface tension.

The three-dimensional version of lattice Boltzmann method D3Q19 [7] on a cubic lattice with nineteen vectors  $\mathbf{c}_k$  of pseudo-particle velocities is realized. The evolution equations for the distribution functions  $N_k$  have the form

$$N_k(\mathbf{x} + \mathbf{c}_k \Delta t, t + \Delta t) = N_k(\mathbf{x}, t) + \Omega_k(N) + \Delta N_k, \quad (5)$$

where  $\Delta t$  is the time step,  $\Omega_k$  is the collision operator, and  $\Delta N_k$  is the change of the distribution functions due to the action of the internal and external body forces.

The hydrodynamic variables (the density  $\rho$  and the velocity  $\mathbf{u}$  of fluid) in a node are calculated as

$$\rho = \sum_{k=0}^b N_k \quad \text{and} \quad \rho \mathbf{u} = \sum_{k=1}^b \mathbf{c}_k N_k. \quad (6)$$

The collision operator is usually used in BGK form

$$\Omega_k = (N_k^{eq}(\rho, \mathbf{u}) - N_k(\mathbf{x}, t)) / \tau, \quad (7)$$

where  $\tau$  is the dimensionless relaxation time.

The Exact Difference Method (EDM) [8] is used for the implementation of the body forces (internal and external forces) in the LBM:

$$\Delta N_k(\mathbf{x}, t) = N_k^{eq}(\rho, \mathbf{u} + \Delta \mathbf{u}) - N_k^{eq}(\rho, \mathbf{u}), \quad (8)$$

where the value of the velocity after the action of the total force  $\mathbf{F}$  at a node is equal to  $\mathbf{u} + \Delta \mathbf{u} = \mathbf{u} + \mathbf{F} \Delta t / \rho$ . The corresponding equilibrium distribution functions  $N_k^{eq}$  are calculated as

$$N_k^{eq}(\rho, \mathbf{u}) = \rho w_k \left( 1 + \frac{\mathbf{c}_k \mathbf{u}}{\theta} + \frac{(\mathbf{c}_k \mathbf{u})^2}{2\theta^2} - \frac{\mathbf{u}^2}{2\theta} \right). \quad (9)$$

To simulate boundaries between liquid and gas, the internal forces acting between neighbour nodes of fluid were introduced by Shan and Chen [9]. For an equation of state of fluid in the form  $P(\rho, T)$ , the total force acting on a node was introduced by Qian and Chen [10] as a gradient of the pseudopotential  $\mathbf{F} = -\nabla U$ , where  $U(\rho, T) = P(\rho, T) - \rho \theta$ . The appropriate isotropic finite difference approximation of gradient operator was proposed in [11]. We use the quite simple well-known “bounce-back” rule to implement the no-slip boundary conditions at the solid electrodes in the LBM simulations. The periodic boundary conditions are used in  $x$  and  $y$  directions.

The level of wettability of solid electrodes by fluids is simulated by interaction forces acting on a node  $\mathbf{x}$  belonging to the fluid from the nearest nodes  $\mathbf{x} + \mathbf{e}_k$  representing the solid boundaries [12]

$$\mathbf{F}_k = w_k \psi(\rho(\mathbf{x})) B(\mathbf{x} + \mathbf{e}_k) \mathbf{e}_k. \quad (10)$$

### 4. Generation of chain of bubbles in dielectric liquid

The flows arising in a dielectric liquid with bubbles due to the action of an electric field were simulated by the lattice Boltzmann method [13,14]. The dielectric liquid with a dissolved gas is considered. The second set of distribution functions  $g_k(\mathbf{x}, t)$  is used for gas (the second component). The Graphics Processing Units (GPUs) are exploited for computer simulations.

A chain of small bubbles is placed in liquid between the flat electrodes (figure 5a). The van der Waals equation of state with possible phase transitions is used.

For a complete description of the process, it is necessary to consider the system of equations of electrohydrodynamics [2]. Together with the hydrodynamic description of the fluid flows by LBM, one should take into account the transfer of charge carriers  $q_i$  with fluid as well as their effective

macroscopic mobility  $b_i$  in an electric field. The total electric charge density is equal to  $q = \sum_i q_i n_i$ , where  $n_i$  are the concentrations of carriers of electric charge. In the case of identical and constant diffusion coefficients for all carriers of electric charges  $D_i = D$ , the following equations are valid

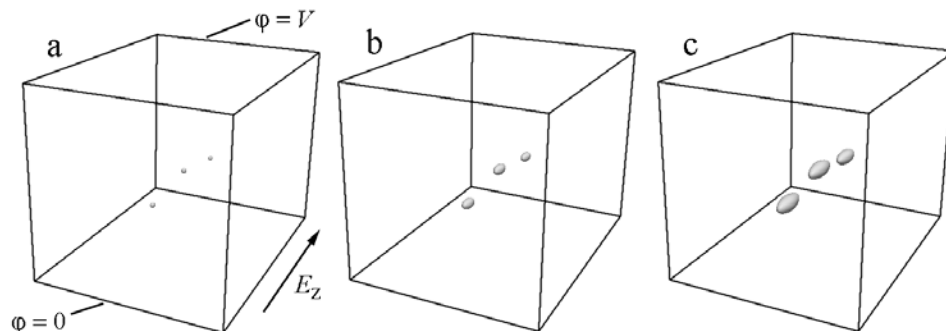
$$\mathbf{j} = q\mathbf{u} - D\nabla q + \sigma\mathbf{E}, \quad (11)$$

$$\frac{\partial q}{\partial t} + \text{div}(q\mathbf{u}) = D\Delta q - \text{div}(\sigma\mathbf{E}), \quad (12)$$

that should be used instead of equations (3). Here,  $\mathbf{u}$  is the velocity of fluid. The local conductivity of material  $\sigma = \sum_i b_i |q_i| n_i$  depends on the local concentrations of charge carriers and, generally, can vary in space and time. In our simulations, the value of the electrical conductivity  $\sigma$  was assumed constant for simplicity and distinct from zero only in the process of breakdown of the gas inside the bubble. Generation and recombination of charge carriers in the volume of dielectric fluid in this study were not considered. The body force acting on a fluid in electric field has the form

$$\mathbf{F} = q\mathbf{E} - \frac{E^2}{8\pi} \nabla \varepsilon + \frac{1}{8\pi} \nabla \left( E^2 \rho \frac{\partial \varepsilon}{\partial \rho} \right). \quad (13)$$

In this paper, the simulation was limited by the moment of the first partial discharge in one of the bubbles. Up to this moment, no free charges are present in the fluid, and the charges transfer equation (12) may be omitted. In this case, the effect of electric field on the substance is limited by second and third terms on the right side of the equation (13).



**Figure 5.** Bubbles growth in strong electric field before generation of the first partial discharge.  $t = 700$ (a),  $800$ (b),  $900$ (c). Lattice is  $192 \times 192 \times 256$ .

Figure 5 shows the gas phase only, the liquid phase is considered to be transparent. After applying a voltage  $V$  to the electrodes, the expansion of existing bubbles occurs (figures 5b,c). This is because of the electrostriction forces [13,14] as well as diffusion of the dissolved gas into the bubbles. An inception of micro-breakdown in one of the bubbles is determined by the electric field inside bubble and by the value of parameter  $Pd$  (Paschen's law). Later, the elongated bubbles merge and form the approximately cylindrical gas–vapor channel.

## 5. Conclusion

The possibility of a "relay-race" wave propagation mechanism of partial discharges in a linear chain of gas inclusions is shown. The lattice Boltzmann method is successfully implemented for three-dimensional computer simulations of flows of dielectric fluid with bubbles. Growth and elongation of bubbles in a liquid dielectric under the action of a strong electric field are simulated. The physical model of propagation of partial discharges along a chain of gas bubbles in a liquid is formulated.

## Acknowledgments

The study was supported by the Russian Scientific Foundation (grant No. 16-19-10229).

## References

- [1] Wu K, Suzuoki Y and Dissado L A 2004 The contribution of discharge area variation to partial discharge patterns in disk-voids *J. Phys. D: Appl. Phys.* **37**(13) 1815–23
- [2] Kupershtokh A L, Karpov D I, Medvedev D A, Stamatelatos C P, Charalambakos V P, Pyrgioti E C and Agoris D P 2007 Stochastic models of partial discharge activity in solid and liquid dielectrics *IET Sci. Meas. Technol.* **1**(6) 303–11
- [3] Karpov D I and Kupershtokh A L 1998 Models of streamers growth with "physical" time and fractal characteristics of streamer structures *Conf. Record 1998 IEEE Int. Symp. on Electrical Insulation, IEEE No. 98CH36239* vol 2 (Arlington, USA) pp 607–10
- [4] Babaeva N Yu, Tereshonok D V, Naidis G V and Smirnov B V 2016 Initiation of breakdown in strings of bubbles immersed in transformer oil and water: String orientation and proximity of bubbles *J. Phys. D: Appl. Phys.* **49**(2) 025202
- [5] Kupershtokh A L, Charalambakos V, Agoris D and Karpov D I 2001 Simulation of breakdown in air using cellular automata with streamer to leader transition *J. Phys. D: Appl. Phys.* **34**(6) 936–46
- [6] Peschot A, Poulain C, Bonifaci N and Lesaint O 2015 Electrical breakdown voltage in micro- and submicrometer contact gaps (100 nm – 10  $\mu$ m) in air and nitrogen *IEEE Holm Conference on Electrical Contacts* pp 280–6
- [7] Qian Y H, d'Humières D and Lallemand P 1992 Lattice BGK models for Navier – Stokes equation *Europhys. Lett.* **17** 479–84
- [8] Kupershtokh A L 2010 Criterion of numerical instability of liquid state in LBE simulations *Comput. Math. Appl.* **59** 2236–45
- [9] Shan X and Chen H 1993 Lattice Boltzmann model for simulating flows with multiple phases and components *Phys. Rev. E* **47** 1815–9
- [10] Qian Y H and Chen S 1997 Finite size effect in lattice-BGK models *Int. J. Mod. Phys. C* **8** 763–71
- [11] Kupershtokh A L, Medvedev D A and Karpov D I 2009 On equations of state in a lattice Boltzmann method *Comput. Math. Appl.* **58** 965–74
- [12] Kupershtokh A L, Ermanyuk E V and Gavrilov N V 2015 The rupture of thin liquid films placed on solid and liquid substrates in gravity body forces *Comm. Comp. Phys.* **17**(5) 1301–19
- [13] Kupershtokh A L and Medvedev D A 2006 Anisotropic instability of a dielectric liquid in a strong uniform electric field: Decay into a two-phase system of vapor filaments in a liquid *Phys. Rev. E* **74**(2) 021505
- [14] Kupershtokh A L 2014 Three-dimensional LBE simulations of a decay of liquid dielectrics with a solute gas into the system of gas-vapor channels under the action of strong electric fields *Comput. Math. Appl.* **67**(2) 340–9