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MODELLING OF PREBREAKDOWN PHENOMENA USING CELLULAR AUTOMATA

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Abstract: In the present work we present two different models developed for the simulation of prebreakdown phenomena in liquid dielectrics and long air gaps. In both cases extensions of classic Cellular Automata were used. In the case of gas dielectrics two different states of conductive structures were introduced, corresponding to streamer and leader channels. Cellular Automata with stochastic transformation rules were used. In the case of liquid dielectrics, the cellular automaton with anisotropic structure factor was used instead of the electric field calculation.

Keywords: cellular automata, computer simulation, breakdown process.

1. Introduction

Breakdown in solid, liquid, and gaseous dielectrics are of great importance in designing power systems, because it defines the limitation of the insulation in cables, transformers, electrical rotating machinery etc. Breakdown is very complex physical process and it depends on dielectric material, electrode configuration, applied voltage etc. In small gaps the application of some critical voltage has as a result the formation of streamers, which are conductive channels of low conductivity. These channels propagate inside the gap and the breakdown occurs when these channels reach the opposite electrode.

In long air gaps the physical process which lead to a breakdown is considerably different. The main characteristic of the discharge is that the conductive tree can propagate in low electric field by reproducing self-sustained conditions across the gap during its propagation. Above some critical voltage the discharge process initiates with the formation of the streamer channels. Streamers propagate over some distance until local electric field in front of them reduces to some value below critical.

Another phenomenon is taking place simultaneously. Due to the current flowing along the streamer channels, there is an increase of the temperature because of Joule heating. Above 1500°K thermal detachment of negative ions enhances the conductivity and lowers the internal field, leading to the formation of the leader. The leader channel propagates along the gap with the leader corona (streamer channels) in front of its tip [1, 2].

In this paper, we present two different models, developed for the simulation of the breakdown process. The first model simulates the breakdown of liquid dielectrics concerning small gaps [3] and the second simulates the breakdown in filled with gas long gaps [4]. Each model is taking into account the different physical mechanisms involved in each case and the computer realization was made using Cellular Automata.

2. Cellular Automata

Cellular Automata (CA) are models developed for the simulation of the evolution of complex natural systems. They can be applied in modeling of physical systems where space and time are discrete and interactions are only local. CA first introduced by von Neumann [5] and despite their structural simplicity may exhibit complex dynamic behavior.

Briefly a cellular automaton consists of:

- A lattice of cells. Each cell can be in one of a finite number of distinct states at each moment of time. The lattice may be two or three-dimensional and of arbitrary size.
- Transformation rules from one state to another depending only on the states of neighborhood cells.

The state of a cell at the next iteration $t+1$ is computed according to some function F which is a function of the states of neighboring cells at time moment t. Thus, in the case of a two dimensional CA, the local rule for the determination of the state of the cell $S_{i,j}$ at time moment $t+1$ is of the form:

$$S_{i,j}^{t+1} = F\left(S_{i-1,j-1}^{t}, \ldots S_{i,j}^{t}, \ldots S_{i+1,j+1}^{t}\right) . \quad (1)$$

The neighborhood of the cell i,j is illustrated in **fig. 1**. In practice CA is a computer program in which:

- A matrix is created with specific element values (integer, real, etc.).
- A function or a set of functions, used to change the values of the matrix elements, is defined.

- The function is applied repeatedly to the matrix, each time changing the values of all the matrix elements simultaneously.

For the development of the models presented in this paper, we used an extension of classic CA, where the CA transformation rules uses not only the states of the cells but also the values of some physical parameters in local neighborhood of a given cell, like electrical field potential. Thus the local rule for the determination of the state of the cell $S_{i,j}$ at time moment $t+1$ is of the form:

$$S_{i,j}^{t+1} = F\,(S^t, p)\ , \qquad (2)$$

where S^t are the current states of CA and p is set of physical parameters in this and neighbor cells. This modification of CA is necessary because the transition of a cell from one state to another is strongly depended on physical parameters like local electric field.

i-1,j+1	i,j+1	i+1,j+1
i-1,j	**i, j**	i+1,j
i-1,j-1	i,j-1	i+1,j-1

Figure 1. The neighborhood of the (i, j) cell.

3. The models

3.1 Breakdown in liquid dielectrics

Extensive experimental data, like statistical time lag between application of the voltage over the gap and the start of breakdown, the asymmetric shape of streamer structures, etc, indicate the importance of stochastic processes in the breakdown of liquid dielectrics. Several authors related the probability of streamer growth with a function of local electric field [6, 7]. The distribution of the electric field potential was calculated by solving the Laplace equation in a region between the electrodes outside of a conductive structure that was considered to be equipotential.

Our study was based on an initially uniform electric field. For fast nanosecond breakdowns in liquid dielectrics, the charge relaxation along the conductive branches is not complete. Thus, the local electric field ahead of the streamer tips is greater than the initial electric field but does not exceed it several times. The last circumstance makes it possible to use the cellular automaton with anisotropic structure factor instead of exact calculation of the electric field [3].

It is assumed that the growth process is described approximately by the following criterion for the growth of a new streamer segment

$$E > E_* - \delta\ , \qquad (3)$$

where E is the electric field potential at each lattice point, and E_* is a characteristic parameter of the material. A random value δ is assumed to take into account inhomogenities of the dielectric, thermal and other fluctuations, the statistical nature of the excited states of the vibrational degrees of the molecules, fluctuations of local microfields acting on the molecules etc. The criterion for the streamer growth (3) is known as Field Fluctuation Criterion (FFC).

The probability density for fluctuations δ is the following:

$$f(\delta) = \frac{\exp(-\delta / g)}{g}\ . \qquad (4)$$

The physical meaning of g is the characteristic width of the distribution (4) of random variable δ.

Local electric field E depends on the applied voltage and the geometry of the gap and also on the local configuration of the conductive structure in the neighborhood of the point in question. The local electric field is calculated from a table of fundamental cell configurations of the states in neighbor cells:

$$E = k \cdot E_0\ , \qquad (5)$$

where E_0 is the initial electric field. The fundamental cell configurations are illustrated in **fig. 2.**

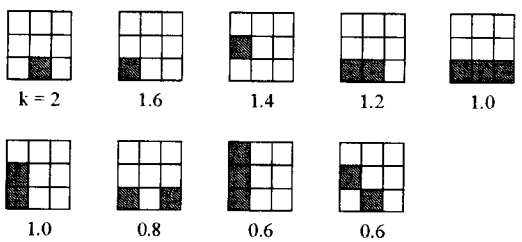

Figure 2. Anisotropic cellular automaton. The central cell becomes conducting if $E>E_*-\delta$, where $E=kE_0$.

Using this method it is possible to calculate the electric field in an easy way without solving the Laplace or the Poisson equations.

After the initial conditions were specified, the system evolves in time. The cellular automaton produces one or another configuration of conducting structure by Monte Carlo method.

3.2 Breakdown in gaseous dielectrics

Breakdown of gases in the case of small gaps can be handled by the same way as it has been applied for liquid dielectrics. The same criterion for the growth of the streamers (FFC) can be used, changing of course the values

of the parameters. The electric field can be calculated by solving either Laplace or Poisson equations.

The breakdown process in long air gaps is considerably different than small gaps. The first is the growth of streamers in front of the leader tips and the second is the streamer-to-leader transition. Thus a new model was developed, taking into account two physical processes, which occur during the discharge.

The leader structure was considered to be equipotential because of its high conductivity, while the branches of streamers have very small conductivity and practically don't influence the electric field potential distribution. Thus, the absence of complete charge relaxation along the branches was modeled by neglecting streamer conductivity. In this case the electric field potential outside the highly conductive structure can be obtained by solving Laplace equation with boundary conditions on electrodes and highly conductive structure.

Two assumptions were made for the growth of the streamer. Firstly the growth is stochastic in time and secondly the probability of a streamer formation is proportional to some function of local electric field $r(E)$, depending on properties of the dielectric. This function is closely complied with the velocity of streamer tip propagation in local electric field E in front of it i.e. $u(E) = h\, r(E)$ [8].

For the streamer growth the FFC was used as in the case of breakdown in liquids. The difference was that the local electric field was calculated by solving the Laplace equation. However, it was necessary to develop a criterion for streamer-to-leader transition. For the transformation of a streamer to highly conductive arc, it is of importance the energy input inside a streamer segment. When in some filament the energy release due to current achieve sufficient value, this filament transforms into a highly conductive arc.

If we consider a small segment of the streamer as a cylinder with height h, cross-section s, and the conductivity σ (very small value), then the energy, which is released in a time interval t, will be

$$W_i = h \cdot s \cdot \sigma \int_{t_i}^{t_i+t} E(t)^2 dt, \qquad (6)$$

where t_i is the moment of time when this bond arose.

So, if the energy released is larger than some critical value, a new highly conductive segment is formed. This means that the criterion of the formation of a new highly conductive segment could be

$$A \int_{t_i}^{t_i+t} E_i^2 dt > W_* \ , \qquad (7)$$

where W_* is some critical value of energy released, and $A = h \cdot s \cdot \sigma$.

For the development of the model we propose to consider that the change of electrical conductivity along the branches can be approximated as a sequence of low conductive (streamer) and highly conductive phases. Taking this into account, the space between the electrodes is divided into cells. Each cell occupied by dielectric can be in three states. The first state is initial one. It means that in this cell nothing has happened. The second state corresponds to the formation of the streamer in this cell. The third state denotes that the transition of the "streamer" state to the highly conductive state has occurred.

Before the initiation of the breakdown, all cells of the dielectric were in the initial state (S0). Then, according to the stochastic criterion for streamer growth (3), some of the cells can turn into the next state (S1) that corresponds to the streamer. According to the model for the streamer transformation to a highly conductive phase, some of the cells can transform from state S1 to states S2 or S4 that corresponds to the cells of downward and upward leaders, respectively. It was assumed that each type of leader is equipotential.

At every time step only the cells, which are contiguous to the electrode surfaces or to streamer or to the highly conductive phase could change state (from S0 to S1). On the other hand only the cells which are contiguous to the electrode surfaces or to the highly conductive phase structure could change their state from S1 to S2 or S4. The procedure proposed was repeated until the conductive tree approaches the opposite electrode.

We used the distinct states of the cellular automaton not only to describe cells that initially were in the dielectric state but also to indicate cells that belong to the upper and lower electrodes (the states S3 and S5).

Such formalization simplifies significantly the logical structure of the computer program. It allows us to consider all cells in a standard way. For example, in PASCAL language, we use the following simple iterative procedure during solution of the Laplace equation:

```
case S[i,j] of
     0,1: Fn[i,j]:= (F[i+1,j] F[i-1,j] F[i,j+1] F[i,j-1])/4;
     2,3: Fn[i,j]:=Fi0;
     4,5: Fn[i,j]:=0;
end;
```

where the letter **F** represents the electric-field potential φ and **Fn** is the values of electric field potential at next iteration step. The configuration of electrodes was defined initially by assigning values "3" or "5" to some elements $S_{i,j}$ of our array of cellular automaton states.

Of course, in reality, the conductive tree consists of the small linear segments, rather than of square elements. To describe this, we introduced a local coordinate system in the vicinity of each cell. The first digit denotes the x coordinate of the neighbor cells (column number), and the second digit denotes the y coordinate of it (row number). Hence, each neighbor cell has a unique value. Thus, this information was kept in the cell as an additional set of states of the cellular automaton to indicate one of the neighbor cells, from which the new conductive bond originated. In this case, it is possible to use this information, for example, to draw segments of the conductive structure, and to consider the structure as a graph consisting of conductive bonds (**fig. 5**).

4. Calculations

The parameters of FFC model in the calculations of fast breakdown in liquid in uniform electric field were E_*=1 and g=0.08. The problem was solved in the rectangular area. Typical result of simulation is shown in **fig. 3.**

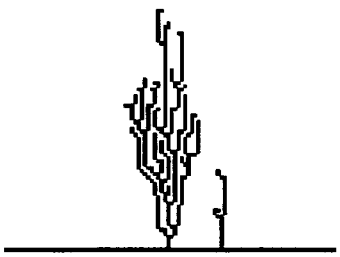

Figure 3. Example of simulation of fast breakdown in liquid in uniform electric field. The electric field was E_0=0.54, E_*=1, g=0.055. Lattice size was 100×100, t=97.

The simulation of breakdown in air gaps was carried out in the rectangular area on lattices up to 200×200.

The problem was solved for model geometry in that breakdown occurs between two electrodes. The bottom electrode was at the electric potential $\varphi = 0$ and the upper one at $\varphi = V_0$, where V_0 is the applied voltage. Periodic boundary conditions in the x direction were used. The mean initial electric field in the gap was $E_0 = V_0/d$, where d is the gap length.

The electric field in the region outside of the highly conductive structure was calculated at every time step by solving the Laplace equation

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0 \qquad (8)$$

with the boundary conditions on electrodes and highly conductive structure.

At every time step new streamers may arise from the tips of existent conductive structure and the transition of one or more streamers to highly conductive phase may occur. This procedure continues until the conductive tree approaches the opposite electrode.

Eight permissible directions (including diagonals) of streamer propagation was used at each site of a square lattice to diminish the anisotropy of the growing structure. The special procedure was used to delay the growth of diagonal bonds to the next time step with the fixed probability $p=2-\sqrt{2}=0.586$. At this value the mean streamer propagation velocity in diagonal direction

$$<u>=\left(\sqrt{2}h(1-p)+\frac{\sqrt{2}h}{2}p\right)r(E)=hr(E). \quad (9)$$

Hence, streamer propagation velocity was ensured equal for all bonds including diagonals, provided that the projection of mean electric field to the corresponding direction was the same.

The reliable physical or experimental data must be used to choose the complementary set of scales for space, time and electric field (or voltage). These three scales can be defined in each particular case that is to be simulated. To emphasize the common features of the phenomena, in this study some arbitrary units are used for length, time, charge, voltage, and electric field.

5. Results

In every calculation the statistical time lag of the conductive tree origin was observed. At fixed geometry it sharply depends on the voltage difference.

The growing structure consists of many individual streamers that propagate in a competitive way. Some of them subsequently transformed into the highly conductive phase. A short pulse of current accompanied each event, because of the stepwise change of full charge of conductive structure.

In **figs. 4** and **5** the results of the simulations of a breakdown in long air gap is illustrated, for point-plane geometry. The parameters of FFC model in the calculations were E_*=1, g=0.08, and E_0=0.3, 0.2.

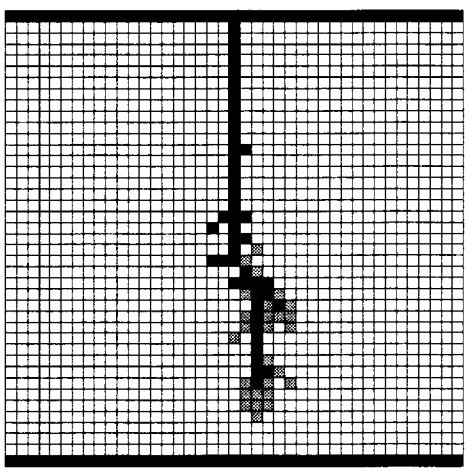

Figure 4. Simulation of a discharge in long air gap. The initial mean electric field was E_0=0.3. Lattice size was 40×40, t=1120.

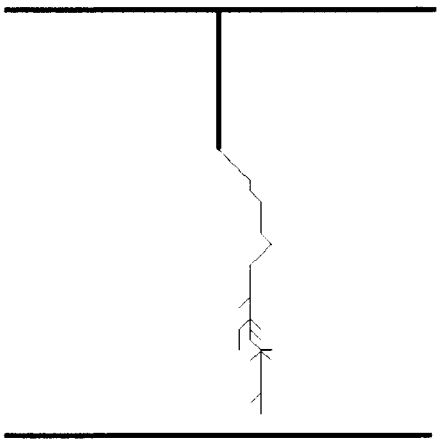

Figure 5. Simulation of a discharge in long air gap with initial mean electric field E_0=0.2, t=1450.

As it was expected with a reduction of mean electric field there is also a reduction of the mean propagation velocity of the conductive structure.

6. Conclusions

The stochastic models of conductive tree growth proposed here, describe main features of breakdown in gaseous and liquid dielectrics, such as statistical time lag, random place of origin, asymmetry and non-reproducibility of conductive structures etc. With a careful calibration of their parameters they can be used mainly for the determination of the breakdown voltage for various shapes of applied voltages and different gap configurations.

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