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Simulation of the Development of Branching Streamer Structures in Dielectric Liquids with Pulsed Conductivity of Channels

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Abstract—A model describing the development of branched streamer structures in dielectrics is proposed, in which the conductivity of each segment of discharge channels has a pulsed character. Calculations performed show that the stochastic model of streamer growth with allowance for the pulsed conductivity of channels describe qualitatively the main features of streamer structure development in liquid dielectrics.

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As is known, breakdown in a liquid dielectric arises upon shorting of the interelectrode gap by a streamer structure developed in the medium. Conditions (applied voltage, discharge gap geometry, pressure, etc.) under which the discharge takes place determine the characteristics of the growth of such streamer structures, including their shape, propagation velocity, current, electric field strength in front of the streamer tip, conducting channel radius, the state of substance in the channels, and the degree of ionization. Bazelyan and Raizer [1] justified the hypothesis about self-consistent variation of the propagation velocity, channel tip radius, and local electric field strength at the streamer tip. Niemeyer et al. [2] originally suggested that the growth probability (i.e., the propagation velocity) at each tip of the streamer structure is related to the local electric field strength in front of the streamer tip under the fixed external conditions (pressure, temperature, etc.). Within the framework of this approach, several criteria of the development of streamer structures were proposed [3–5], which have been examined in [6, 7].

The stochastic models of streamer structure development proposed in [2, 4–6, 8] provided a qualitative description of the complicated spatial patterns of discharge, the main stochastic laws of their initiation and formation, and some other phenomena. The parameters of streamer structures can be, in principle, calculated by solving the self-consistent problem of the growth of conducting channels with allowance for the charge transfer, the electric field redistribution in the discharge gap, and hydrodynamic flows in the system.

The conductivity of a channel is determined by the degree of ionization, which, in turn, depends on the energy released, the density and composition of the discharge plasma, and the energy losses for the channel expansion and radiation. However, simultaneous calculations of the plasma composition and (especially) conductivity in numerous channels of the developing streamer structure at each moment of its evolution, with allowance for the inhomogeneous distribution of characteristics in the cross section of each channel, is now practically impossible. The computational difficulties considerably increase in view of a large difference in scales of the diameters of streamer channels (~10 μm) and the interelectrode gaps (from ~1 mm to ~10 cm). The results of previous calculations performed for the streamers with constant conductivity showed that the average channel conductivity influences both the propagation velocity and the geometry of branched streamer structure [6, 9].

It is also necessary to take into account that, during the streamer structure development in liquid dielectrics, the electric current in the circuit has a pulsed component [10, 11]. The current pulses are correlated with the pulses of light emitted from the entire streamer structure or its parts. The emission pulses are separated by rather long intervals of time. This circumstance implies in fact that the charge transfer along the conducting channels can proceed in a pulsed manner, being related to microdischarges in some elements of the channels, so that the conductivity of channels possesses a pulsed temporal character and a localized spatial character. During the streamer structure development, the overall conductivity in the streamer channels can be relatively low, except for the moments when microdischarges take place in some elements of the channels.

Previously, Noskov et al. [12] proposed a model describing the development of streamers, in which the charge transfer in the entire structure proceeded by means of partial discharges in the elements of the structure. However, that model did not include equations describing the relaxation of charges. Therefore, the microdischarges were assumed to be instantaneous, which implies infinitely large local conductivity.

This Letter presents a model describing the dynamics of streamers in a liquid dielectric with allowance for a pulsed character of the conductivity and the relaxation of charges in each element of the channel. The model is formulated with neglect of the distributions of plasma temperature and composition in the channel cross section, as well as of the hydrodynamics of channel expansion.

Let us consider a streamer structure comprising a system of thin conducting elements, which represent parts of discharge channels. New elements of the channels are formed when the local field strength in the dielectric exceeds a certain critical value (E_*). The propagation and branching of the channels are described using the Field Fluctuation Criterion of growth [3, 6], according to which a new conducting element appears at a perimeter of the streamer structure network within one time step τ, provided that

$$E_i > E_* - \delta_i, \tag{1}$$

where E_i is the projection of the local electric field strength on the direction in which the new ith element is formed and δ_i is a random quantity that takes into account the fluctuations caused by local inhomogeneities of the critical field E_* in the liquid and the fluctuations of microscopic fields in the dielectric relative to the average local field E_i. The critical field E_* decreases with increasing duration of the time step. An analogous relationship exists between the "electrical strength" of a liquid and the duration of voltage application (voltage–time characteristic).

Experiments demonstrate a strong dependence of the breakdown probability on the electric field strength in the discharge gap. For this reason, fluctuations δ are assumed to obey the exponential probability distribution:

$$f(\delta) = \frac{1}{g}\exp(-\delta/g). \tag{2}$$

In the range of prebreakdown fields, a determining role in providing condition (1) is played by rather large δ values representing statistically rare events corresponding to the "tail" of the distribution curve. The characteristic scale of fluctuations δ is provided by the width g of the distribution function. The parameter g and the function $E_*(\tau)$ describe the characteristic dynamic electrical strength of a particular dielectric.

The probability of formation of a new element of the structure of length h during the time step τ according to Eqs. (1) and (2) was considered in [7, 9]. This analysis gives the following dependence of the average velocity of propagation of the streamer tip as a function of the local electric field strength:

$$v(E) = A\exp(E/g), \text{ where } A = \frac{h}{\tau}\exp(-E_*/g). \tag{3}$$

Note that the propagation velocity is not directly related to the streamer tip radius, since the latter value also should correspond to the local field strength E [1].

The electric field $\mathbf{E}$ and potential φ in the discharge gap at each time step are calculated using the equations of electrostatics as

$$\operatorname{div}(\varepsilon\mathbf{E}) = 4\pi\rho, \quad \mathbf{E} = -\nabla\varphi, \tag{4}$$

where ε is the dielectric constant and ρ is the charge density. The charge transfer along branches of the streamer structure is described by the following equations:

$$\frac{\partial\rho}{\partial t} + \operatorname{div}\mathbf{j} = 0, \quad \mathbf{j} = \sigma\mathbf{E}, \tag{5}$$

where $\mathbf{j}$ is the current density in the discharge channel and σ is the specific conductivity of the channel. The conductivity of the dielectric is assumed to be zero.

A microdischarge in an element of a channel in the existing branched streamer structure arises when the electric field in this element obeys the condition

$$E > E_{\text{ign}} - \tilde{\delta}, \tag{6}$$

where E_{ign} is the characteristic microdischarge ignition field and $\tilde{\delta}$ is a random quantity that takes into account fluctuations. The latter quantity is also assumed to obey an exponential probability distribution $f(\tilde{\delta}) = \exp(-\tilde{\delta}/\tilde{g})/\tilde{g}$. Once a microdischarge takes place, the conductivity of the given element of the channel acquires the constant value Λ. The change in the channel radius with time is ignored. After the microdischarge ignition, the channel remains conducting until the electric field strength in the given element drops below a certain critical value E_{cr}, after which the microdischarge terminates and the channel conductivity vanishes. The validity of the conditions of streamer structure growth (1) and the microdischarge ignition (6) and termination are checked at each time step in all elements of the structure.

The problem formulated above was solved on a cubic lattice in a three-dimensional region with 50 × 50 × 50 nodes. The streamer growth was initiated with a protrusion on one (point) of the two electrodes. The opposite flat electrode was separated by distance d from the point. The coefficient of field inhomogeneity was equal to ~7.6. Convenient dimensionless units were introduced using the corresponding field (E_*), length (h), and time ($\tau_0 = h/\Lambda$) scales. In these terms, the other parameters of the model are expressed as $g/E_* = 0.05$, $E_{\text{ign}}/E_* = 0.25$, $E_{\text{cr}}/E_* = 0.033$, $\tilde{g}/E_* = 0.023$, $\tau/\tau_0 =$

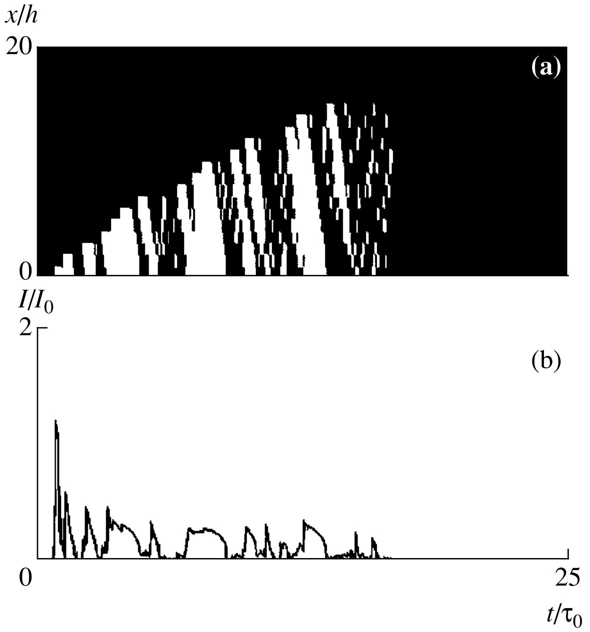

Fig. 1. The results of numerical simulation illustrating the growth of a linear streamer channel: (a) an *x–t* diagram of the conductivity evolution in a single linear channel (white color indicates the conducting (light-emitting) segments of the channel); (b) the time variation of current in the circuit ($V/V_0 = 2.3$).

2.5×10^{-4}, and $d/h = 28$. The scales of current and voltage are expressed in terms of $I_0 = \Lambda E_* h$ and $V_0 = E_*/h$, respectively.

Figure 1 shows the results of numerical simulation of a straight channel arising at the point electrode. The *x–t* diagram (Fig. 1a) shows that a conducting region can appear in any part of the channel where the electric field strength is sufficiently high. The charge transfer in conducting elements leads to a decrease in the field strength in this part of the channel and, hence, to an increase in the field strength in the adjacent nonconducting elements. As a result, microdischarges take place in the neighboring parts of the channel and the conductivity waves propagate in both directions along the channel. It was repeatedly reported (see, e.g., [10, 11]) that "light emission waves" propagated along channels of the streamer structure. It is natural to assume that these emitting regions correspond to the zones of energy release and, hence, of increased conductivity.

Every microdischarge in a certain element of the channel produces polarization of this element, which is accompanied by a current pulse in the external circuit (Fig. 1b). For the elements situated far from electrodes, this event corresponds to a current pulse of low intensity. Current pulses of high amplitude correspond to discharges in the channel elements adjacent to the electrodes.

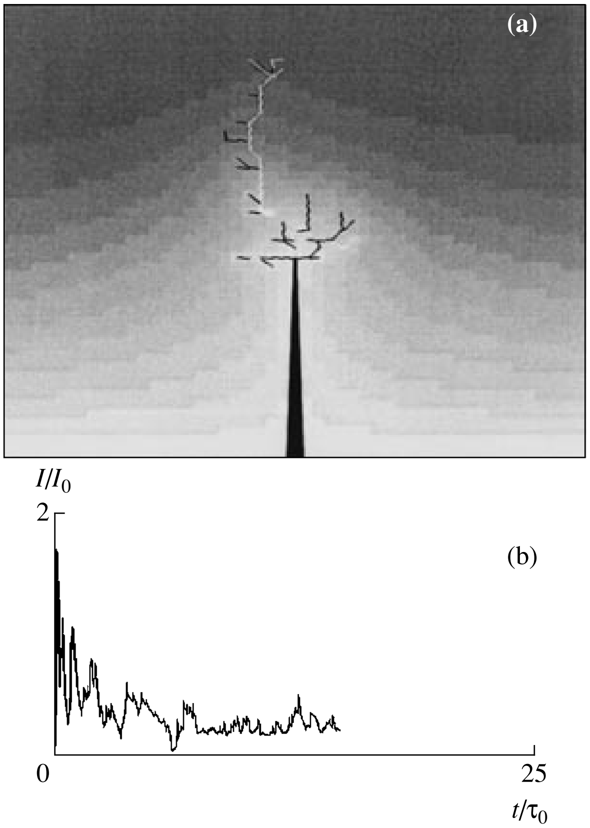

Fig. 2. The results of numerical simulation illustrating the development of a branched streamer: (a) a typical streamer structure (gray levels indicate the potential distribution in the central cross section of the discharge gap; black and white elements correspond to nonconducting and conducting branches, respectively); (b) the time variation of current in the circuit ($V/V_0 = 3.0$).

The main features of propagation of the conductivity waves along discharge channels also remain valid for the development of branched structures (Fig. 2a). At each time step, most branches occur in the nonconducting state (indicated by black color). Nevertheless, the structure continuously develops due to the branches in which energy release takes place from time to time (as indicated by white color). For a sufficiently high repetition rate of current pulses with finite duration, a continuous current component appears in the external circuit (Fig. 2b). In experiment [13], branches where the energy release does not take place (current pulses are absent) for a rather long time can decompose into fragments and disappear even despite voltage applied to the entire structure. The results of our calculations correspond to the development of rather fast streamer structures, in which the process of decay of channels can be ignored. In this case, the growth of a streamer structure is maintained due to the conductivity waves arising in individual branches of the structure.

The results of our simulations agree well with the observations of pulsed electric discharge in liquid dielectrics. The proposed model explains the pulsed character of light emission and the streamer current variation observed in experiment.

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