02CH37319

Proceedings of 2002 IEEE 14 th International Conference on Dielectric Liquids ICDL 2002

Graz, Austria
July 7 - 12, 2002

Sponsored by
IEEE - Dielectrics and Electrical Insulation Society

Modeling of Electrohydrodynamic Flows and Micro-bubbles Generation in Dielectric Liquid by Lattice Boltzmann Equation Method

D. A. Medvedev, A. L. Kupershtokh

Lavrentyev Institute of Hydrodynamics SB RAS,
Lavrentyev prosp. 15, Novosibirsk 630090
RUSSIA

***Abstract*:** The Lattice Boltzmann Equation (LBE) method was used to simulate a liquid flow with space electric charge. New approach to describe the charge transfer due to convective charge transport was proposed. Simulations of two-dimensional electrohydrodynamic flow showed pulsations caused by the injection of charged blobs and by the hydrodynamic instability of charged jet. Generation and growth of micro-bubbles in strong electric field near the electrode surface was simulated by the LBE method with interparticle interaction. A threshold electric field was found, below that the bubble generation does not occur. At higher fields, the time of bubble development t_d is approximately inversely proportional to the square of the voltage.

INTRODUCTION

Method of the lattice Boltzmann equation (LBE) is extensively used in the last decade to simulate flows of viscous liquids [1]. Method LBE is based on the mesoscopic kinetic equation for the simple model system. The kinetic nature brings certain advantages over conventional finite-difference methods, such as improved stability, easy handling of complex geometries, parallel computation and efficient multiphase simulations.

Recently, a modification of the LBE method was developed in order to incorporate electohydrodynamic (EHD) effects: charge injection, convective and conductive electric charge transport, and action of electrodynamic forces on charged liquid [2]. EHD flows in different geometries were simulated.

In this paper, we propose further extension of the model and its new applications.

Computations of two-dimensional EHD flow were performed. Liquid conductivity was supposed to be zero, different injection conductivity was considered. The flow has strong oscillations, caused at the initial stage by injection of charge blobs that screen the injecting electrode and hinder further injection. At later stage, oscillations are caused principally by the instability of the main flow mode, similar to instabilities predicted in [3].

In strong electric field, especially near sharp tips and edges on the electrode surface, the negative pressure is produced that can be high enough for the formation of vapor bubbles. Computations clearly revealed this process, confirming the availability of this mechanism. Following breakdown of gas inside bubbles can lead to the breakdown of dielectric liquid itself.

In the present work the time, length, charge, voltage, electric field and conductivity are given in arbitrary units.

LATTICE BOLTZMANN EQUATION METHOD

The LBE method is based on solving the kinetic equation for a certain model system in which special particles can move along the links of a fixed lattice. Particle velocities can have values from the limited set only, so that each particle moves exactly to one of neighbor nodes in one time step. The basic value of velocity of the particles in the LBE method is $c_1 = h/\Delta t$, where Δt is the time step, h is the length of lattice bond. The evolution equations for one-particle distribution functions N_k have form

$$N_k(\mathbf{x}+\mathbf{c}_k, t+\Delta t) = N_k(\mathbf{x}, t) - (N_k - N_k^{eq})/\tau.$$

Second term in the right-hand side of this equation is the collision operator, corresponding to the collision integral in the Boltzmann equation. In present work, we used the BGK form of collision operator, that is the relaxation to local equilibrium and is common in the lattice Boltzmann method [1]. The equilibrium distribution functions N_k^{eq} depend on the local density $\rho = \sum N_k$, flow velocity at a node $\mathbf{u} = \left(\sum \mathbf{c}_k N_k\right)/\rho$, and temperature, so that the conservation laws for mass, momentum, and energy are satisfied locally, i. e., $\sum N_k^{eq} = \rho$, $\sum \mathbf{c}_k N_k^{eq} = \rho\mathbf{u}$, and $\sum (\mathbf{c}_k - \mathbf{u})^2 N_k^{eq} = \rho(Td + u^2)$ where d is the dimension of space. The relaxation time τ governs the transport coefficients: viscosity, heat conductivity, and diffusivity ($1/2<\tau<\infty$).

In computations we used a one-dimensional model with three values of particle velocity $c_k = -1$, 0, and +1, and a two-dimensional model on a square lattice with three velocity values $|\mathbf{c}_k| = 0$, 1, and $\sqrt{2}$ (9 possible velocity vectors). Hereinafter all values are in lattice units, for which $\Delta t = 1$ and $h = 1$. For both models the temperature is constant $T = 1/\sqrt{3}$, and the kinematic viscosity is $\nu = (\tau - 1/2)/3$.

For simulation of bubble generation in electric field, the LBE method with interparticle interaction [4] was used. This method allows one to model the dynamics of multiphase and multicomponent fluids. If interparticle attraction is sufficiently strong, two phases of different densities can coexist in certain range of initial density (rarefied gas-like phase and

0-7803-7350-2/02/$17,00 © 2002 IEEE

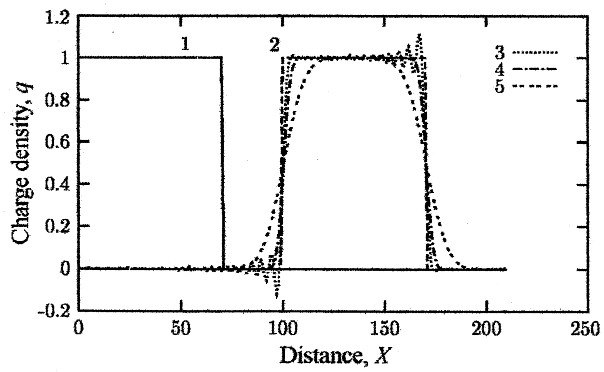

Fig. 1. Transformation of charge distribution in a one-dimensional liquid flow in the case of zero electrical conductivity. Computations with the method of additional LBE-component. Velocity of uniform flow was u = 0.1. Initial charge distribution (curve *1*); theoretical charge distribution without diffusion (*2*) at t = 1000. Computed charge distribution at time t = 1000 for the diffusivity $D_3 = 3.3 \cdot 10^{-4}$ (*3*); $D_3 = 3.3 \cdot 10^{-3}$ (*4*), and $D_3 = 3.3 \cdot 10^{-2}$ (*5*).

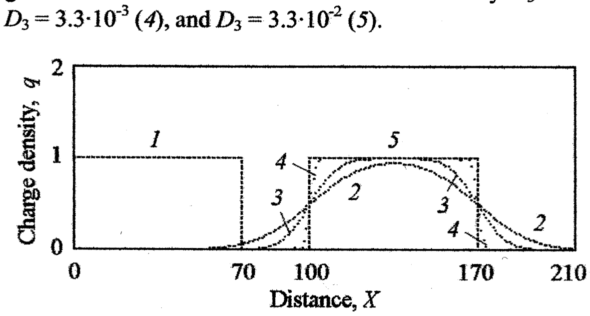

Fig. 2. Transformation of charge distribution in a one-dimensional liquid flow in the case of zero electrical conductivity. Velocity of uniform flow was u = 0.1. Initial charge distribution at time t = 0 (curve *1*). Method of "LBE-particles", $D_1 = h^2/6\Delta t$ (*2*); method of mean velocity, $D_2 = 0.045\, h^2/\Delta t$ (*3*); additional LBE-component method for $D_3 = 0.0033\, h^2/\Delta t$ (*4*); and theoretical charge distribution without diffusion (5) at time t = 1000.

dense liquid-like phase). The coexistence curve is similar to that of the Van-der-Waals fluid, and value of interparticle attraction define the temperature of phase transition [4].

COMPUTATION OF THE CONVECTIVE CHARGE TRANSFER

In [2], two methods of computing the convective charge transfer were developed: 1) the method of "LBE-particles", and 2) the method of mean velocity. The first one has the numerical diffusivity $D_1 = h^2/6\Delta t$. The diffusivity for the second method is $D_2 = |\mathbf{u}|(h/\Delta t - |\mathbf{u}|)/2$ which depends on flow velocity and in general is essentially lower than its maximal value $D_2|_{\max} = h^2/8\Delta t$, achieved at $|\mathbf{u}| = h/2\Delta t$.

In present work, we propose a new method based on using the additional LBE component with zero mass. Its evolution equation is written as

$$f_k(\mathbf{x}+\mathbf{c}_k, t+\Delta t) = f_k(\mathbf{x},t) - (f_k(\mathbf{x},t) - f_k^{eq}(\mathbf{x},t))/\tau_q .$$

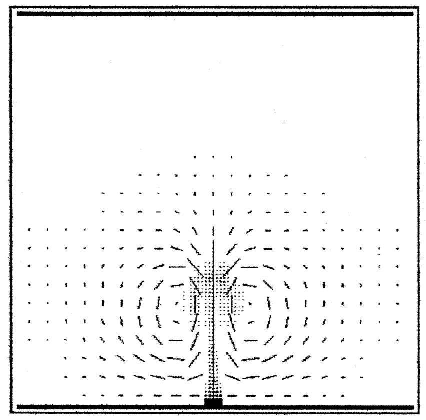

Fig. 3. Velocity field (shown by lines) and charge density (shown by shades of gray) in 2D EHD flow. Time t = 510.

Distribution functions of the additional component, f_k^{eq} depend on the charge density in the node $q = \sum f_k^{eq}$ and on the velocity $\mathbf{u}$ of the main flow (additional component has zero momentum). For this method, the charge diffusivity is $D_3 = \dfrac{h^2}{3\Delta t^2}\left(\tau_q - \dfrac{1}{2}\right)$. It depends on the relaxation time τ_q, and can be made sufficiently low for the τ_q close to 1/2. Computation results for different values of D_3 are shown in Fig. 1. The flow of liquid was uniform with periodic boundary conditions. The mass velocity of the liquid was constant $u = u_0$ and equal to 0.1. The electric charge was initially distributed as $q(x) = q_0$ at $x_1 < x < x_2$.

Figure 2 presents the comparison of results of the previously proposed methods (curves 2 and 3), and the new method for the computation of the convective charge transfer (curve 4). The flow of liquid and the initial charge distribution were the same as for Fig. 1. Thus, the method of the additional LBE-component allows one to reduce numerical diffusivity by more than order of magnitude in comparison with the mean velocity method. For lower values of D_3, oscillations of charge density arose (see Fig. 1, curve 3).

INSTABILITIES OF EHD FLOW

In [2], a simulation of 2D EHD flow in the blade-plane geometry was carried out.

Computations were performed in the rectangular region between two horizontal plane electrodes. Periodic boundary conditions in the X direction were used. Electric potential of the upper electrode was $\varphi = 0$, and of the lower electrode $\varphi = \varphi_0 = 106$, so the mean electric field between electrodes was 1. There was a small protrusion 5×2 lattice sites in the middle of the lower electrode. The sites, which are contiguous to this protrusion, were slightly conductive (the conductivity was $\sigma = 2 \cdot 10^{-4}$).

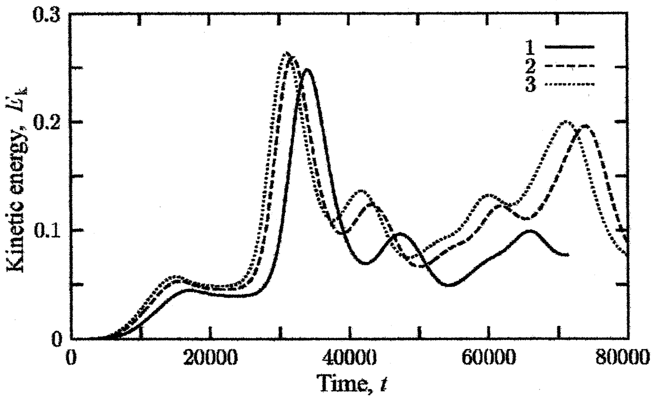

Fig. 4. Time dependence of flow kinetic energy. Conductivity $\sigma_0 = 10^{-4}$ (*1*), $2{\cdot}10^{-4}$ (*2*), $3{\cdot}10^{-4}$ (*3*).

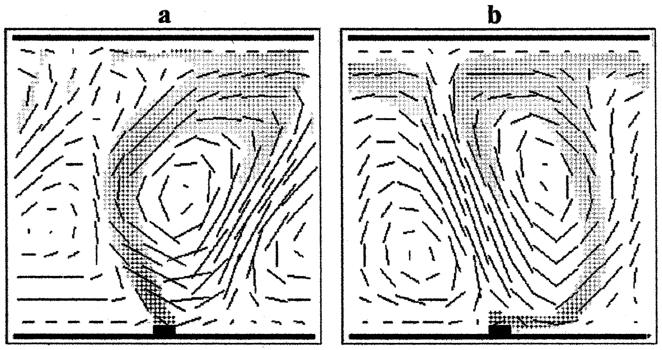

Fig. 5. Velocity field (shown by lines) and positive charge density (shown by shades of gray) at late stages of 2D EHD flow. Time $t = 3.9\ 10^4$ (a), $t = 7.2\ 10^4$ (b)

A viscous flow in the form of a plane vortical dipole moving to the upper electrode was clearly observed. The velocity field and charge distribution at some moment of time are shown in Fig. 3.

In computations of [2], no charge sink existed, hence, charge accumulation and flow cessation should occur later.

In the present work, in the same geometry, a conducting layer existed also near the upper electrode with the same conductivity σ_0 as the layer near the blade. Time evolution of the flow kinetic energy E_k for different values of σ_0 is shown in Fig. 4. Flow pulsations are readily observable. Figure 5 presents the velocity field and the charge distribution at the late stage of flow development. One can see complicated flow pattern with several vortices and shifted and distorted charged jets. Hence, the instability of the main flow mode predicted in [3] is observed.

SIMULATION OF BUBBLE GENERATION AND GROWTH

One of the breakdown mechanisms is the so called "bubble" mechanism. Micro-bubbles of gas or vapor may pre-exist

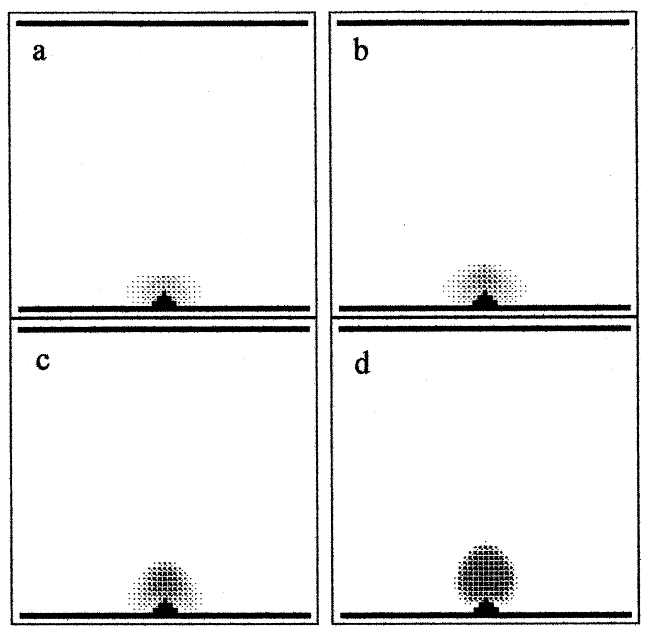

Fig. 6. Initiation and growth of vapor bubble in strong electric field on the electrode surface. Time $t = 80$ (a), 100 (b), 120 (c), 140(d).

in liquid, or they can be generated after the application of high electric voltage. The cause of bubble formation may be local heating and evaporation of liquid (the thermal mechanism, that was thoroughly investigated in [4]).

An alternative to the thermal mechanism is the homogenous nucleation of bubbles in the region of low (or negative) pressure. Such region can exist near sharp tips and edges on the electrode surface, where electric field is high enough. This mechanism can be named electrodynamic cavitation, it was theoretically studied in [6]. Once nucleated, bubbles will grow, and the breakdown of gas inside them will occur at certain critical size. The breakdown of bubbles leads to field enhancement near the apex of bubble and may cause the breakdown of dielectric liquid itself.

To simulate this phenomenon, the LBE model with interparticle interaction was used [4]. Initial density in computations was chosen corresponding to the high-density (liquid) phase. The interparticle attraction was sufficiently strong to enable phase transition. The permittivity of liquid ε was considered constant and independent on density (this is possible, if $\varepsilon \approx 1$). Hence, the electrostrictive forces were not taken into account in this work.

Figure 6 presents different stages of the formation and growth of vapor bubble due to electrodynamic cavitation. The density inside the bubble decreased by three orders of magnitude.

The voltage dependence of the bubble growth time is shown in Fig. 7. The average electric field is $E_a = V/L$, where V is the applied voltage, L is the interelectrode distance. The time lag t_d was measured between voltage application and

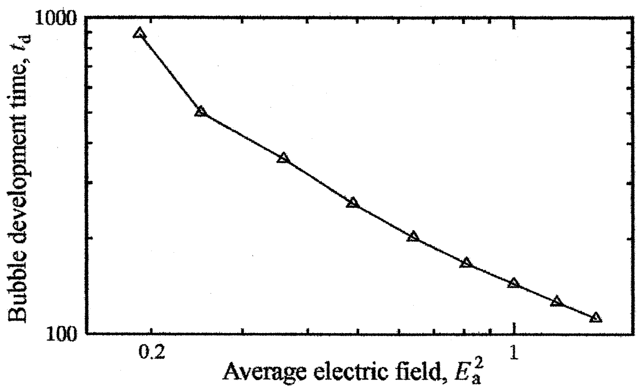

Fig. 7. Electric field dependence of the bubble development time.

generation of a bubble of certain size ($R \approx 5$ lattice units). The results show that there is a threshold electric field, below that the bubble generation does not occur. At higher fields, the development time is approximately inversely proportional to the square of the average electric field E_a ($t_d \sim E_a^{-2}$). The same dependence of the development time on the energy release $j \cdot E$ is mentioned in [4] for the breakdown of liquid argon and water ($t_d \sim (j \cdot E)^{-1}$). In our case, j is proportional to E, hence, the energy release is proportional to E^2. The same voltage dependence can be derived from [7], there $R(t) \sim (E^2 t)^{2/3}$, hence, for fixed R this leads to $t_d \sim E^{-2}$.

CONCLUSION

A new extension of the lattice Boltzmann method for EHD simulations is developed. The method of the convective charge transport computation with an additional LBE-component allows one to reduce the numerical diffusivity by more than one order of magnitude.

The simulations of 2D EHD flow revealed its pulsed behavior primarily due to the instability of the main flow mode.

For the first time, generation and growth of vapor bubbles in high electric field near the electrode was observed in simulations. Bubbles are generated by the electrodynamic cavitation mechanism. The voltage dependence of bubble growth time is obtained, the results agree with available theoretic and experimental data.

The method developed is very promising for simulation of variety of EHD phenomena including early stages of the electric breakdown.

REFERENCES

[1] Chen, S., Doolen, G. D. "Lattice Boltzmann method for fluid flows", Annu. Rev. Fluid Mech., 1998, Vol. 30, pp. 329–364.

[2] Medvedev, D. A., Kupershtokh, A. L. "Use of the Lattice Boltzmann Equation Method to Simulate Charge Transfer and Electrohydrodynamic Phenomena in Dielectric Liquids". Proc. of the 2nd Int. Workshop on Electrical Conduction, Convection, and Breakdown in Fluids, Grenoble, France, 2000. P. 60–63.

[3] Vázquez, P. A., Pérez, A. T., Castellanos, A., Atten P. "Dynamics of electrohydrodynamic laminar plumes: Scaling analysis. Phys. Fluids", 2000, Vol. 12, No. 11, pp. 2809–2818.

[4] Shan, X., Chen, H. "Lattice Boltzmann model for simulating flows with multiple phases and components", Phys. Rev. E., 1993, Vol. 47, No. 3, pp. 1815–1819.

[5] Jones, H. M., Kunhardt, E. E. "Development of pulsed dielectric breakdown in liquids", J. Phys. D: Appl. Phys., 1995, Vol. 28, No. 1, pp. 178–188.

[6] Krasuzki, Z. "Breakdown of liquid dielectrics", Proceedings of Royal Society, 1966, Vol. A294, No. 1438, pp. 393–404.

[7] Watson, P. K., Chadband, W. G., Sadeghzadeh-Araghi, M. "The role of electrostatic and hydrodynamic forces in the negative-point breakdown of liquid dielectrics", IEEE Trans. Electr. Ins., 1991, Vol. 26, No. 4, pp. 543–559.