AN APPROACH TO FRACTAL DIMENSION OF LIGHTNING PATTERN

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Abstract: In the present paper the fractal dimension of lightning patterns is calculated. The lightning patterns were generated using a stochastic model, which was developed, in High Voltage Laboratory of the University of Patras. Two different phases of plasma in the channels of the conductive structure were considered, the low conductivity phase (streamer) and the high conductivity phase (leader). Two different methods for the estimation of fractal dimension were used, namely the Box Counting Method and the Method of Correlation Function.

Keywords: Simulation, Fractal Dimension, Stochastic Modeling.

1. INTRODUCTION

Niemeyer, Pietronero and Wiessmann [1] presented the first stochastic model for the simulation of dielectric breakdown phenomena. Since then several other models have been developed in order to simulate prebreakdown phenomena in gaseous, liquid and solid dielectrics [1-6]. These models are able to generate patterns that they are more or less close to real observed conductive structures. The conductive structure propagates from one electrode to the other in a stepwise manner with the help of a stochastic growth criterion. Usually the growth criterion is depended on the local electric field i.e. $p \nabla E$.

Very early it was found out that the generated structures, as well as the real observed, had fractal characteristics. The name fractal was coined by Mandelbrot, from the Latin word "fractus", to described objects that were too irregular to fit into traditional geometrical settings. Thus several authors tried to calculate the fractal dimension of the patterns created by stochastic models and, sometimes, of real observed conductive structures [1-6].

For the calculation of the fractal dimension several methods, like Box Counting Method, Correlation function Method, radius of gyration, tip radius and axial extension have been used. Due to their different definition these methods may give different results even for the same structure. In the present paper two methods for the estimation of the fractal characteristics were used. The first was the, rather easy but very popular, Box Counting Method and the second was the Method of Correlation Function.

A stochastic model specially developed for the simulation of the lightning and the breakdown of long gaps generated the lightning patterns. This model has originally presented in ICLP2000- [7] and latter enhanced and completed in [8]. The main innovation of the model is the consideration of two different states of channels, which corresponds to streamers and leaders. The gradual changes of conductivity along the branches are approximated as a stepwise transition from the state of plasma in the channels of low conductivity (streamers) to a phase of much higher conductivity.

2. MODELING Of LIGHTNING PROCESS

As it was already mentioned the stochastic model has two different states of plasma, corresponding to streamer and leader. It was assumed that the streamers do not influence the distribution of electric field because of their low conductivity. On the other hand the leader was considered to be equipotential due to its high conductivity. A new streamer bond is added to the structure if the criterion

$$E_i > E_* 4 \iota \quad (1)$$

is fulfilled. Here E_i is the local electric field, E_* is a parameter of the model, which depends on the dielectric and ι is a random value, which is assumed to take into account inhomogenities of the dielectric, thermal and other fluctuations, including fluctuations of local microfields acting on the molecules and also uncertainties due to the action of external conditions (for example atmospheric ionization, air density, humidity etc.). In this paper we use an exponential probability density for fluctuations ι.

The physical mechanism of the streamer to leader transition is not very clear although several theories have been proposed [9-12]. At least for the positive leader it seems that that it is of importance the energy release due to the current flow inside streamer filaments. We used a somewhat simplified approach. If we consider a small segment of the steamer as a cylinder with height *h*, cross section *S* and conductivity ω (very small value), then the total energy released by time *t* is

$$W_i = h \cdot S \cdot \omega \int_{t_i}^{t} E(t)^2 dt \qquad (2)$$

where t_i is time when this bond arose.

Thus, if the released energy is greater than a certain critical value, a new highly conductive segment is formed. It should be noted that we do not considered how some "reverse" processes such as light emission, hydrodynamic expansion etc, influence the streamer dynamics. Nevertheless, we introduced some restriction on the condition of the streamer to leader transition. We assume that in the channel the local electric field must exceed a certain minimal field, in order to ensure sufficient energy release and, hence, to prevent the decay of plasma by these "reverse" processes.

The simulation was carried out in a rectangular area with mesh size of 128x128. The conductive structure begins to grow from the tip of a rod, which represents the starting point of the lightning. The electric field is calculated by solving the Laplace equation

$$\frac{\partial^2 \pi}{\partial x^2} + \frac{\partial^2 \pi}{\partial y^2} = 0 \qquad (3)$$

with boundary conditions on the electrodes and the leader structure. The electric potential of the plane is $\lambda=0$ and the potential of the electrode, as well as the leader, is equal to $\lambda=\lambda_0$ (arbitrary unit). At every time step new streamer segments are added to the structure, while some of the existent streamers are converted to leader. Eight permissible directions (including diagonals) of streamer propagation were allowed at each site of the lattice to diminish the anisotropy of the growing structure.

3. ESTIMATION OF THE FRACTAL DIMENSION

Several methods have been introduced to estimate the fractal dimension of structures of different nature [5, 13]. In the present work two different methods were applied: Box Counting Method and the Method of Correlation Function. In our opinion these are more preferable for the calculation of fractal dimension of patterns generated in a rod – plane geometry (lack of central symmetry).

In Box Counting Method a tree pattern is covered by a set of square lattices of different lattice space [14]. In the case of a tree dimensional object a set of cube lattices should be used. The number of lattice boxes occupied by a compact figure, for example a square or a circular disk, is, $N(h) \propto 1/h^d$, where *h* is the lattice spacing and *d* is the Euclidean dimension. If the structure is a fractal one then

$$N(h) \propto h^{-D} \qquad (4)$$

where *D* is the fractal dimension that is less than *d*. This means that for example an object with fractal dimension between 1 and 2 is larger than 1-dimensional (having infinite length) and smaller than 2-dimensional (having zero area). The fractal dimension can be obtained as the absolute value of the slope of the curve *ln(N)* plotted versus *lnh*. If the curve is close to a straight line in a limited range of value *h* then this structure has fractal characteristics only in this range of scales.

The second method that we used for the estimation of the fractal dimension of our conductive patterns was the Method of Correlation function [13]. The correlation function *C(r)* was calculated for all possible vectors of shift ***r*** using the distribution density over all the structure. Then the angle averaging was carried out to obtain the correlation function *C(r)* as a dependence of absolute value of shift. For fractal structures the correlation function satisfies the formula

$$C(r) \propto 1/r^{d-D} \qquad (4)$$

4. RESULTS

In all the simulations, arbitrary units for voltage, space and time were used. In all the calculations the parameter E_* of the dielectric was equal to 1. The Correlation Function Method was tested using the set of diffusion-controlled clusters up to 10000 particles specially obtained in two-dimensional space as it was described in detail in [15]. The value of the fractal dimension averaged over 4 clusters was *D=1.67* that is in good agreement with the value *D=1.68* obtained in work [15].

However, for patterns of not so large size, the log-log plot in the method of correlation function has no range in which it is really straight line (even for compact figure). In this case, we can obtain only an approximate value of slope of the straight line in some reasonable range of the scale using the least square method with some fixed initial and final points. The points of log-log plots were initially averaged over 10 conductive structures that were obtained at the same values of parameters E_0 and *W*. In our work for the estimation of the fractal dimension we selected the points from 3 to 15 for Box Counting Method and the points from 3 to 10 for Correlation Function Method (**fig. 1**). Fractal characteristics of the conductive structures were investigated for 4 different sets of parameters. The results are shown in Table 1.

The Box Counting Method gave values of fractal dimension that varies from 1.10 to 1.32. Values of fractal dimension close to 1.0 were expected because almost all patterns consist of one or two leaders only. In general, the

Table 1: Estimates of fractal dimension of conductive structures for different sets of the parameters E_0 and W_*

Mean electric field E_0	Critical energy W_*	Runs	Counting-box method	Correlation function method
0.15	50	10	1.10	1.02
0.15	5	10	1.19	1.10
0.25	50	10	1.16	–
0.25	5	10	1.32	–

value of fractal dimension increases when the initial electric field E_0 increases and the critical energy for the streamer to leader transition decreases. If we set the critical energy equal to zero we obtain the ordinary Laplace model.

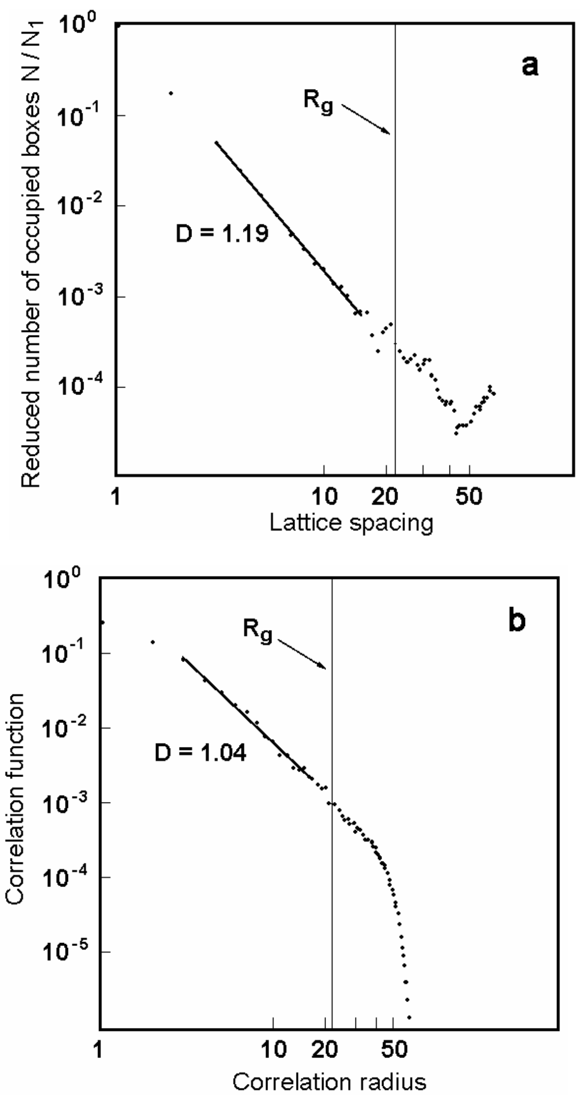

Fig. 1: The linear approximation of the log-log plot for Box Counting Method (a) and for Correlation Function Method (b).

For the two last sets of parameters (Table 1), the plot of logarithm of correlation function averaged over ten conductive structures is not linear as a function of *ln r*. One can conclude that the method of correlation function is not much applicable to structures that appear at breakdown in lightning or long air gaps when these structures are not of large size.

5. CONCLUSIONS

A two-stage model was used for the simulation of the lightning process. It is also possible to use the model for the simulation of breakdown in long gaps. A disadvantage of the model is that we can use only arbitrary units for space, voltage and time.

Two methods were used for the calculation of fractal dimension of lightning patterns. Both methods after averaging over 10 structures gave values close to 1.0. This is not surprising because the electric field in front of the conductive structure is not so large and the structures not so branched. However, for individual patterns the fractal dimension may vary in a wide range.

The estimation of fractal dimension could be a useful tool for the comparison of the results obtained from different stochastic models, especially for the simulation of breakdown in small gaps where the fractal characteristics are more intense. It is also possible to compare the fractal dimension of patterns obtained from stochastic models

6. REFERENCES

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