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Stochastic Model of Streamer Growth in Dielectric Liquids with Hydrodynamic Expansion of Streamer Channels

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***Abstract*:** Stochastic model of streamer propagation in dielectric liquids was developed. Simple approximate model is proposed that enables to calculate the pressure in cylindrical channel using the expansion velocity. Three-dimensional model for simulation of streamer growth was realized that takes into account in the first approximation expansion of streamer channels.

INTRODUCTION

Theory of breakdown in dielectric liquids must take into account the regularities of formation of streamer structure under the action of high electric field. Stochastic models developed during last several years simulate growth of streamer structures sufficiently well [1-4]. Nevertheless, all these models show only qualitative agreement with experiments.

It is well known that streamer growth in dielectric liquids is accompanied by series of hydrodynamic phenomena. Some observations such as increase of voltage of streamer initiation at point electrode under elevated pressure, shock waves formation from streamers, breaking up of streamer filaments into strings of bubbles after voltage removal etc allowed some authors to suppose that density of substance in streamer channels is significantly lower than the density of ambient dielectric liquid [5]. The increase of channel diameter influences its conduction and, consequently, the energy balance in channel. Unfortunately, a correct description of hydrodynamic phenomena accompanying propagation of streamers is too complex for exact solution. First, it is required solving partial differential equations inside the region containing both liquid phase and conductive phase of strongly irregular configuration. Second, the characteristic scales of the problem that are diameter of streamer channels and gap length differ greatly (by more than 100 times). These obstacles make exact numerical methods not applicable for three-dimensional simulations. However, this problem could be simplified by using some approximate description of expansion of streamer filament. We proposed a simple approximate expression for calculating the pressure inside the channel using kinematics of its expansion. Three-dimensional computer simulations of streamer structure growth were performed that takes into account expansion of streamer filaments due to energy release in them.

MODEL OF STREAMER GROWTH

Prebreakdown processes in dielectric liquids involve formation of structures of conducting channels called streamers in gap between electrodes. The complete classification of stochastic models of streamer growth with physical time was made in [2]. In present work, the model proposed in [1] and developed in [2, 3] was used. Streamer growth was considered as discrete in space and in time process of joining of new conducting segments to existing structure or electrode in accordance with simple probabilistic rules. Field fluctuation criterion of growth [1] was used in the present work. A new segment is added to conducting structure at the lattice sites where

$$E_i > E_* - \delta_i \,. \tag{1}$$

E_i the is mean value of projection of electric-field strength onto linear segment connecting two lattice sites one of which belongs to conducting structure. An exponential probability distribution for fluctuation δ was used $\varphi(\delta) = \frac{1}{g}\exp(-\delta / g)$ which is equivalent to $\delta = -g\ln(\xi)$. Here ξ is a random number that is uniformly distributed in the interval from 0 to 1.

It was shown in [4] that mean velocity of streamer growth in the case of constant electric field ahead the tip is

$$\upsilon(E) = A\exp(E / g)\text{, with } A = \frac{1}{\tau}\exp(-E_* / g) \;, \tag{2}$$

where τ is the time step.

Electric field distribution in the gap between electrodes were calculated at every time step by solving the Poisson equation and charge conservation law

$$\Delta\varphi = -4\pi q / \varepsilon \;, \quad \frac{\partial q}{\partial t} = -\operatorname{div}\vec{j} \tag{3}$$

with following relations

$$\vec{j} = \sigma \cdot \vec{E} \;, \quad \vec{E} = -\nabla\varphi \;. \tag{4}$$

Here φ is the electric field potential, $\vec{E}$ is the electric field strength, ε is the permittivity, q is the electric charge density, σ is the conductivity of streamer channel, $\vec{j}$ is current density. System (3) – (4) was solved at every time step τ.

STREAMER CHANNEL EXPANSION

As mentioned above, exact description of hydrodynamic expansion of streamer channel is impossible. Therefore, it is needed to find an appropriate approximation of the problem. The following assumptions were made to simplify significantly the problem: 1) streamer structure consists of cylindrical segments of radius R_c that is much smaller than their length l, 2) hydrodynamic expansion of each segment is independent of other segments. This approach allowed us to incorporate

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the dynamics of expansion of streamer channels due to the energy release in them into the stochastic model of streamer growth.

Similar problems of expansion of long cylindrical channel were solved for channel of electric spark in liquids. Self-similar solution was obtained in [6] for constant velocity of channel wall u_c that is valid at linear increase of energy release. The value of pressure inside the channel p_c is constant too and was fitted well in [6] by the following expression

$$p_c - p_0 = 1.35\,\rho_0\,u_c^{\,2}(c_0/u_c)^{0.3}\ , \qquad (5)$$

where p_0 is the hydrostatic pressure, ρ_0 is the mass density, c_0 is the sound velocity of unperturbed liquid. The approximation (5) is valid in the range of velocities $0.05\,c_0 \le u_c \le c_0$.

Actually, expansion of streamer channel is essentially non-stationary problem. Another analytical expression for p_c was obtained earlier in [7] within the framework of the linear acoustic approximation

$$p_c = p_0 + \rho_0\,(R_c\,\dot{u}_c + u_c^{\,2})\ln(2c_0 t/R_c) - \rho_0\,u_c^{\,2}/2\,. \qquad (6)$$

Nevertheless, this approximation is also not well suitable for non-stationary flows. Moreover, in this approximation the liquid was assumed as non-compressible.

The problem of expansion of infinite cylindrical piston with walls impenetrable for substance was solved by numerical method described in [8]. System of equations includes Euler's and mass conservation law equations. The Tait equation of state for water was used that is valid under pressures up to 3 GPa at $b = 7.15$

$$p - p_0 = \rho_0\,c_0^{\,2}[(\rho/\rho_0)^b - 1]/b\,. \qquad (7)$$

It is well-known that solution for the problem of expansion of cylindrical channel is the flow with shock wave diverging from symmetry axis. At expansion velocities $u_c < 100$ м/c shock wave velocity practically coincides with sound velocity in unperturbed liquid.

Initial conditions in the liquid were $u = 0$, $p = p_0$, $\rho = \rho_0$. The equation of energy balance in the channel was used as boundary condition at channel-liquid interface

$$\frac{N(t)}{l} = p_c\,\frac{ds}{dt} + \frac{1}{\gamma_* - 1}\,\frac{d}{dt}(p_c\,s)\ , \qquad (8)$$

where p_c is the pressure inside the channel, $N(t)$ is energy release into channel at moment of time t, l is the channel length, $s = \pi\,Rc2$ is the channel cross-section, R_c is the channel radius. The equation of state for ideal gas with effective adiabatic exponent γ_* was used for channel plasma. The initial radius of new streamer filament was supposed to be of the order of 1 μm. Lattice size was large enough so that shock wave did not reach external boundary of domain of simulation for whole simulation time.

Numerical solution of electrodynamic problem (3)–(4) gives distribution of electric field and electric currents in streamer branches for each moment of time. This gives energy release $N(t)$ in each branch at time moment t. Then, the problem is the solving of partial differential equations for each segment of streamer structure. Subsequent approximation that gives solution of non-stationary problem of channel expansion for arbitrary function $N(t)$ is given in the next section.

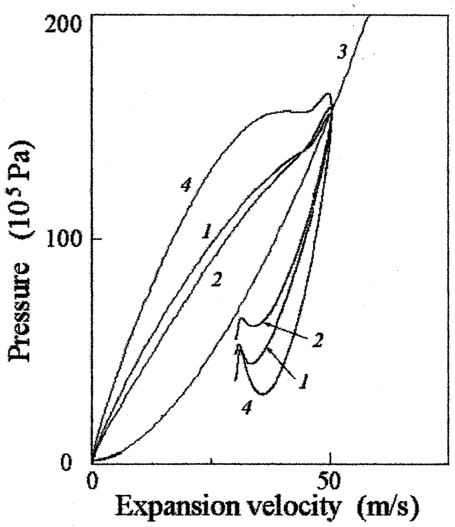

Figure 1. Pressure on cylinder wall in dependence on its expansion velocity. *1* is the exact numerical solution, *2* is the approximation (9) with $k = 1.5$, *3* is the self-similar and *4* is the acoustic solutions.

APPROXIMATE FORMULA FOR PRESSURE INSIDE CHANNEL

Expansion of streamer channels usually is relatively slow because the energy release is rather weak. If wall velocity u_c changes gradually enough, the flow of liquid near the channel has time to be rearranged at each moment in accordance with the new boundary conditions. Hence, the flow there is close to self-similar one. Thus, we proposed the approximate formula in which the term proportional to acceleration of wall was added to the right hand side of (5)

$$p_c = p_0 + 1.35\,\rho_0\,c_0^{\,0.3}\,u_c^{\,1.7} + k\,\rho\,R_c\,\dot{u}_c\ . \qquad (9)$$

Here k is coefficient that can be evaluated by comparing the results obtained using the exact solution with the approximate formula (8) for non-stationary hydrodynamic problem. The value $k = 1.5$ was appropriate for typical flows discussed above.

Up to date experimental data give no information about energy release in segments of streamer branch. Exact hydrodynamic problem was solved for several plausible dependences $N(t)$. Plot of p_c vs. u_c is shown in Fig. 1 (curve *1*) for energy release per unit length of channel in the form $\tilde{N}(t) = N_0 \sin^2(2\pi t/T)$, where $N_0 = 0.5$ W/cm, $T = 0.2$ μs. Curve *2* corresponds to (9). Acoustic approximation (6) and self-similar solutions (5) are also shown (curves *3* and *4*, respectively). Thus, formula (9) approximates the problem of expansion of streamer channel better than self-similar and acoustic solutions.

From (8) and (9) we obtained the system of ordinary differential equations for approximate description of channel expansion

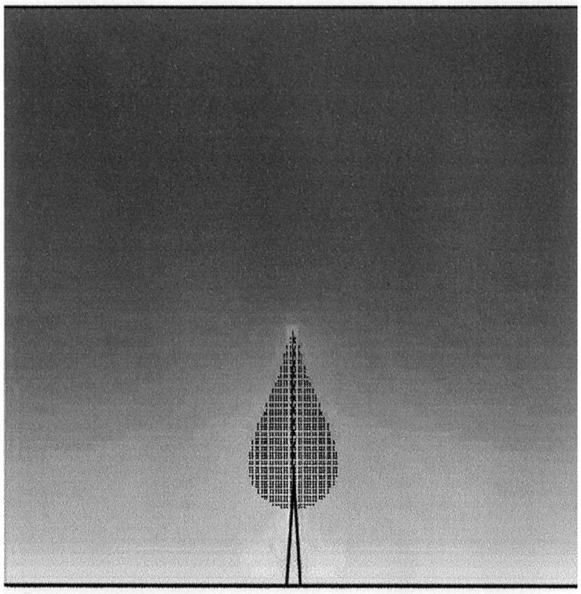

Figure 2. Single linear channel propagation. $V = 30$ kV. $t = 0.27$ μs. Gray levels denote distribution of electric potential in central cross-section of the interelectrode gap. Region of liquid involved in hydrodynamic flow is shown by dots.

$$\dot{u}_c = (A\,p_c - B\,u_c^{1.7})/R_c, \qquad \dot{R}_c = u_c,$$

$$\dot{p}_c = -C\,p_c\,u_c/R_c + D\,N(t), \tag{10}$$

where coefficients $A = 1/(k\rho_0)$, $B = 1.35c_0^{0.3}/k$, $C = 2\gamma$, and $D = (\gamma - 1)/\pi l$ are constant. Thus, the equations (10) allow us to solve non-stationary problem for three-dimensional irregular structures and take into account expansion of streamer channels in stochastic simulations of streamer growth in liquids.

STREAMER GROWTH SIMULATION

Simulations of streamer growth were carried out for point – plane electrode geometry. Numerical method developed in [3] was used for solving the system of equations (3) – (4). Lattice size was 50×50×50. Distance between electrodes was $d = 4$ mm and the lattice distance was $h = 100$ μm. The height of point electrode was equal to 10 lattice units. Initial radius of each new streamer segment was $R_c = 2.5$ μm. The first results presented below were obtained for constant conductivity of streamer segments $\sigma = 0.027$ Ohm^{-1}cm^{-1}. Thus, resistance of a segment was inversely proportional to its cross-section.

The values of parameters A and g in (2) that govern stochastic growth of streamer were obtained by comparing average velocity of simulated structure with experimental data. It was shown that velocity of growth of structure obtained depends on both growth velocity function (2) and conductivity of streamer. Parameters $A = 0.013$ μs^{-1} and $g = 45$ kV/cm provide reasonable propagation velocity of streamer of order of 5 km/s for $V \approx 30$ kV and $d = 4$ mm [5].

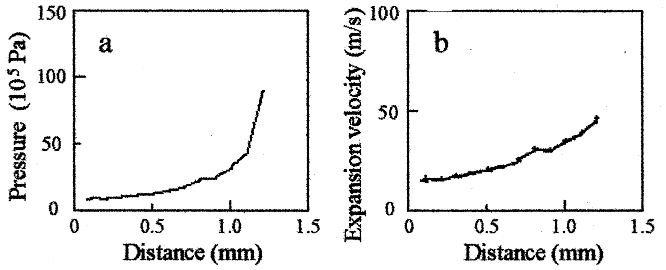

Figure 3. Pressure inside single linear channel (*a*) and its expansion velocity (*b*) in dependence on distance from tip electrode along the channel L. $t = 0.27$ μs.

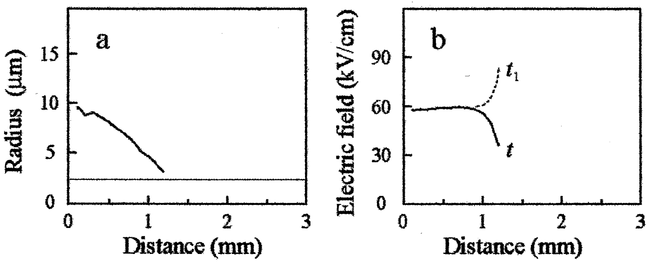

Figure 4. Change of radius of single linear channel from point electrode to streamer tip (*a*) and electric field stress inside the channel (*b*). $t = 0.27$ μs, $t_1 \approx 0.26$ μs.

Simulation of Single Channel Propagation

The typical growth of single linear conducting channel is shown in Fig. 2. Gray levels denote the different values of electric potential in central cross-section of the interelectrode gap. The average velocity of single channel propagation was 4.9 km/s that is about by 3 times greater than sound velocity. Average propagation velocity increases with applied voltage that leads to decrease of angle of Mach cone.

Figures 3 and 4 show change of the pressure, channel radius, velocity of expansion, and electric field strength along the channel at the moment corresponding to Fig. 2. The pressure increases monotonically along the streamer channel and becomes maximal at its top (Fig. 3,*a*). The expansion velocity near the streamer tip was about 50 m/s (Fig. 3,*b*). The radius of the channel increases in time (Fig. 4,*a*). Horizontal line in Fig. 4,*a* shows initial channel radius. The electric-field strength is approximately constant inside the channel except the neighborhood of its tip. After new conducting segment of the channel aroused, the initial electric-field strength inside it near the tip is comparatively high (Fig. 4,*b*, moment t_1). Then, the electric-field strength quickly fell down in this part of the channel due to the charge flow there toward the streamer tip (moment t). The charge distribution became more similar to that as if the point charge placed at the streamer tip.

Simulation of Branching Structure Growth

Projection of typical three-dimensional streamer structure obtained using the stochastic model proposed is shown in Fig. 5. Time dependences of conduction current, maximal size of growing structure (the radius from the pin tip to the most distant point of the tree), and maximal electric-field strength

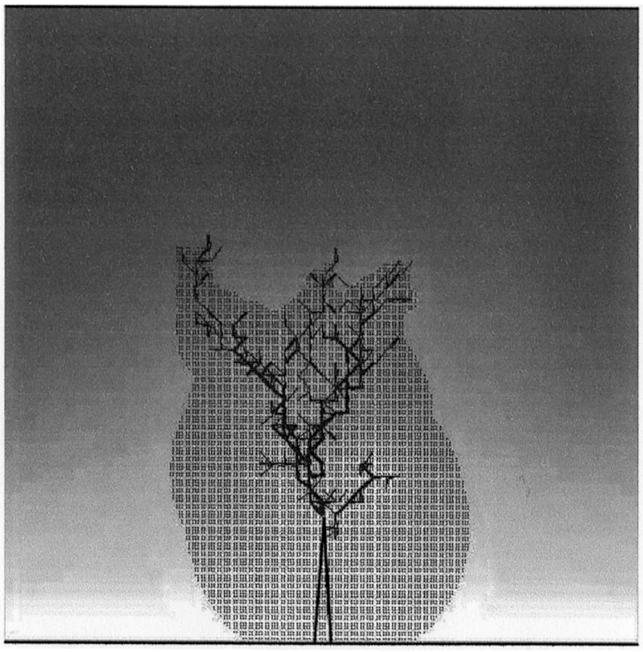

Figure 5. Streamer structure at $V = 24$ kV. $t = 0.74$ μs.

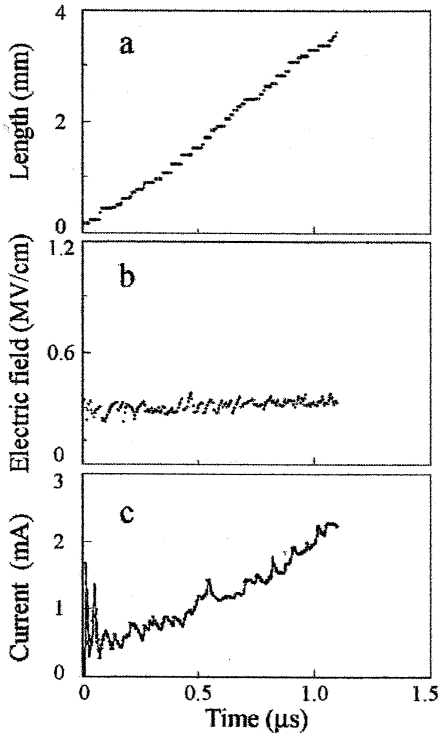

Figure 6. Characteristics of dynamics of streamer. (*a*) maximal size of streamer structure, (*b*) maximal electric-field strength all over ahead the streamer tips, (*c*) conduction current during streamer growth.

all over ahead the streamer tips E_{max} are shown in Fig. 6. The growth velocity of streamer is approximately constant in time and increases with applied voltage. For the structure shown in Fig. 5, the mean growth velocity was 3.4 km/s.

Maximal electric field E_{max} was approximately constant during simulation of growth. Initially the electric field ahead the newly originated conductive element is comparatively low. Then, the electric field increases due to charge relaxation in streamer branch (Fig. 6,*b*). The mean value of E_{max} at which growth occurs during simulation is about by ten times lower than electric field at which streamer initiates from point electrode in experiment [9]. However, this difference could be explained. Indeed, the real value of electric field ahead the streamer head is greater than E_{max} because 1) E_{max} is the mean electric field over lattice distance h and 2) the radius of streamer head ~ 5 μm $\ll h$. Thus, we obtain reasonable agreement between the model and experiment [9] taking into account both these factors.

CONCLUSION

Three-dimensional model for simulations of streamer growth was realized that takes into account in the first approximation expansion of streamer channels. Main electrical and hydrodynamic values are in qualitative agreement with known experimental data.

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